

Rotation of a Rigid Object About a Fixed Axis

CHAPTER OUTLINE

- 10.1 Angular Position, Velocity, and Acceleration
- 10.2 Rotational Kinematics: Rotational Motion with Constant Angular Acceleration
- 10.3 Angular and Linear Quantities
- 10.4 Rotational Energy
- 10.5 Calculation of Moments of Inertia
- 10.6 Torque
- 10.7 Relationship Between Torque and Angular Acceleration
- 10.8 Work, Power, and Energy in Rotational Motion
- 10.9 Rolling Motion of a Rigid Object

ANSWERS TO QUESTIONS

- Q10.1** 1 rev/min, or $\frac{\pi}{30}$ rad/s. Into the wall (clockwise rotation). $\alpha = 0$.



FIG. Q10.1

- Q10.2** $+\hat{\mathbf{k}}, -\hat{\mathbf{k}}$

- Q10.3** Yes, they are valid provided that ω is measured in degrees per second and α is measured in degrees per second-squared.

- Q10.4** The speedometer will be inaccurate. The speedometer measures the number of revolutions per second of the tires. A larger tire will travel more distance in one full revolution as $2\pi r$.
- Q10.5** Smallest I is about x axis and largest I is about y axis.
- Q10.6** The moment of inertia would no longer be $\frac{ML^2}{12}$ if the mass was nonuniformly distributed, nor could it be calculated if the mass distribution was not known.
- Q10.7** The object will start to rotate if the two forces act along different lines. Then the torques of the forces will not be equal in magnitude and opposite in direction.
- Q10.8** No horizontal force acts on the pencil, so its center of mass moves straight down.
- Q10.9** You could measure the time that it takes the hanging object, m , to fall a measured distance after being released from rest. Using this information, the linear acceleration of the mass can be calculated, and then the torque on the rotating object and its angular acceleration.
- Q10.10** You could use $\omega = at$ and $v = at$. The equation $v = R\omega$ is valid in this situation since $a = R\alpha$.
- Q10.11** The angular speed ω would decrease. The center of mass is farther from the pivot, but the moment of inertia increases also.

- Q10.12** The moment of inertia depends on the distribution of mass with respect to a given axis. If the axis is changed, then each bit of mass that makes up the object is a different distance from the axis. In example 10.6 in the text, the moment of inertia of a uniform rigid rod about an axis perpendicular to the rod and passing through the center of mass is derived. If you spin a pencil back and forth about this axis, you will get a feeling for its stubbornness against changing rotation. Now change the axis about which you rotate it by spinning it back and forth about the axis that goes down the middle of the graphite. Easier, isn't it? The moment of inertia about the graphite is much smaller, as the mass of the pencil is concentrated near this axis.
- Q10.13** Compared to an axis through the center of mass, any other parallel axis will have larger average squared distance from the axis to the particles of which the object is composed.
- Q10.14** A quick flip will set the hard-boiled egg spinning faster and more smoothly. The raw egg loses mechanical energy to internal fluid friction.
- Q10.15** $I_{\text{CM}} = MR^2$, $I_{\text{CM}} = MR^2$, $I_{\text{CM}} = \frac{1}{3}MR^2$, $I_{\text{CM}} = \frac{1}{2}MR^2$
- Q10.16** Yes. If you drop an object, it will gain translational kinetic energy from decreasing gravitational potential energy.
- Q10.17** No, just as an object need not be moving to have mass.
- Q10.18** No, only if its angular momentum changes.
- Q10.19** Yes. Consider a pendulum at its greatest excursion from equilibrium. It is momentarily at rest, but must have an angular acceleration or it would not oscillate.
- Q10.20** Since the source reel stops almost instantly when the tape stops playing, the friction on the source reel axle must be fairly large. Since the source reel appears to us to rotate at almost constant angular velocity, the angular acceleration must be very small. Therefore, the torque on the source reel due to the tension in the tape must almost exactly balance the frictional torque. In turn, the frictional torque is nearly constant because kinetic friction forces don't depend on velocity, and the radius of the axle where the friction is applied is constant. Thus we conclude that the torque exerted by the tape on the source reel is essentially constant in time as the tape plays.

As the source reel radius R shrinks, the reel's angular speed $\omega = \frac{v}{R}$ must increase to keep the tape speed v constant. But the biggest change is to the reel's moment of inertia. We model the reel as a roll of tape, ignoring any spool or platter carrying the tape. If we think of the roll of tape as a uniform disk, then its moment of inertia is $I = \frac{1}{2}MR^2$. But the roll's mass is proportional to its base area πR^2 . Thus, on the whole the moment of inertia is proportional to R^4 . The moment of inertia decreases very rapidly as the reel shrinks!

The tension in the tape coming into the read-and-write heads is normally dominated by balancing frictional torque on the source reel, according to $TR \approx \tau_{\text{friction}}$. Therefore, as the tape plays the tension is largest when the reel is smallest. However, in the case of a sudden jerk on the tape, the rotational dynamics of the source reel becomes important. If the source reel is full, then the moment of inertia, proportional to R^4 , will be so large that higher tension in the tape will be required to give the source reel its angular acceleration. If the reel is nearly empty, then the same tape acceleration will require a smaller tension. Thus, the tape will be more likely to break when the source reel is nearly full. One sees the same effect in the case of paper towels; it is easier to snap a towel free when the roll is new than when it is nearly empty.

- Q10.21** The moment of inertia would decrease. This would result in a higher angular speed of the earth, shorter days, and more days in the year!
- Q10.22** There is very little resistance to motion that can reduce the kinetic energy of the rolling ball. Even though there is static friction between the ball and the floor (if there were none, then no rotation would occur and the ball would slide), there is no relative motion of the two surfaces—by the definition of “rolling”—and so no force of kinetic friction acts to reduce K . Air resistance and friction associated with deformation of the ball eventually stop the ball.
- Q10.23** In the frame of reference of the ground, no. Every point moves perpendicular to the line joining it to the instantaneous contact point. The contact point is not moving at all. The leading and trailing edges of the cylinder have velocities at 45° to the vertical as shown.

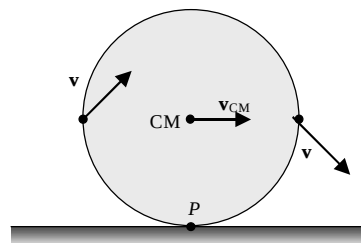


FIG. Q10.23

- Q10.24** The sphere would reach the bottom first; the hoop would reach the bottom last. If each object has the same mass and the same radius, they all have the same torque due to gravity acting on them. The one with the smallest moment of inertia will thus have the largest angular acceleration and reach the bottom of the plane first.
- Q10.25** To win the race, you want to decrease the moment of inertia of the wheels as much as possible. Small, light, solid disk-like wheels would be best!

SOLUTIONS TO PROBLEMS

Section 10.1 Angular Position, Velocity, and Acceleration

- P10.1** (a) $\theta|_{t=0} = \boxed{5.00 \text{ rad}}$
- $$\omega|_{t=0} = \left. \frac{d\theta}{dt} \right|_{t=0} = 10.0 + 4.00t|_{t=0} = \boxed{10.0 \text{ rad/s}}$$
- $$\alpha|_{t=0} = \left. \frac{d\omega}{dt} \right|_{t=0} = \boxed{4.00 \text{ rad/s}^2}$$
- (b) $\theta|_{t=3.00 \text{ s}} = 5.00 + 30.0 + 18.0 = \boxed{53.0 \text{ rad}}$
- $$\omega|_{t=3.00 \text{ s}} = \left. \frac{d\theta}{dt} \right|_{t=3.00 \text{ s}} = 10.0 + 4.00t|_{t=3.00 \text{ s}} = \boxed{22.0 \text{ rad/s}}$$
- $$\alpha|_{t=3.00 \text{ s}} = \left. \frac{d\omega}{dt} \right|_{t=3.00 \text{ s}} = \boxed{4.00 \text{ rad/s}^2}$$
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Section 10.2 **Rotational Kinematics: Rotational Motion with Constant Angular Acceleration**

***P10.2** $\omega_f = 2.51 \times 10^4 \text{ rev/min} = 2.63 \times 10^3 \text{ rad/s}$

(a) $\alpha = \frac{\omega_f - \omega_i}{t} = \frac{2.63 \times 10^3 \text{ rad/s} - 0}{3.2 \text{ s}} = \boxed{8.22 \times 10^2 \text{ rad/s}^2}$

(b) $\theta_f = \omega_i t + \frac{1}{2} \alpha t^2 = 0 + \frac{1}{2} (8.22 \times 10^2 \text{ rad/s}^2) (3.2 \text{ s})^2 = \boxed{4.21 \times 10^3 \text{ rad}}$

P10.3 (a) $\alpha = \frac{\omega - \omega_i}{t} = \frac{12.0 \text{ rad/s}}{3.00 \text{ s}} = \boxed{4.00 \text{ rad/s}^2}$

(b) $\theta = \omega_i t + \frac{1}{2} \alpha t^2 = \frac{1}{2} (4.00 \text{ rad/s}^2) (3.00 \text{ s})^2 = \boxed{18.0 \text{ rad}}$

P10.4 $\omega_i = 2000 \text{ rad/s}, \alpha = -80.0 \text{ rad/s}^2$

(a) $\omega_f = \omega_i + \alpha t = 2000 - (80.0)(10.0) = \boxed{1200 \text{ rad/s}}$

(b) $0 = \omega_i + \alpha t$

$$t = \frac{\omega_i}{-\alpha} = \frac{2000}{80.0} = \boxed{25.0 \text{ s}}$$

P10.5 $\omega_i = \frac{100 \text{ rev}}{1.00 \text{ min}} \left(\frac{1 \text{ min}}{60.0 \text{ s}} \right) \left(\frac{2\pi \text{ rad}}{1.00 \text{ rev}} \right) = \frac{10\pi}{3} \text{ rad/s}, \omega_f = 0$

(a) $t = \frac{\omega_f - \omega_i}{\alpha} = \frac{0 - \frac{10\pi}{3}}{-2.00} \text{ s} = \boxed{5.24 \text{ s}}$

(b) $\theta_f = \bar{\omega} t = \left(\frac{\omega_f + \omega_i}{2} \right) t = \left(\frac{10\pi}{6} \text{ rad/s} \right) \left(\frac{10\pi}{6} \text{ s} \right) = \boxed{27.4 \text{ rad}}$

P10.6 $\omega_i = 3600 \text{ rev/min} = 3.77 \times 10^2 \text{ rad/s}$

$\theta = 50.0 \text{ rev} = 3.14 \times 10^2 \text{ rad}$ and $\omega_f = 0$

$\omega_f^2 = \omega_i^2 + 2\alpha\theta$

$0 = (3.77 \times 10^2 \text{ rad/s})^2 + 2\alpha(3.14 \times 10^2 \text{ rad})$

$\alpha = \boxed{-2.26 \times 10^2 \text{ rad/s}^2}$

P10.7 $\omega = 5.00 \text{ rev/s} = 10.0\pi \text{ rad/s}$. We will break the motion into two stages: (1) a period during which the tub speeds up and (2) a period during which it slows down.

While speeding up, $\theta_1 = \bar{\omega} t = \frac{0 + 10.0\pi \text{ rad/s}}{2} (8.00 \text{ s}) = 40.0\pi \text{ rad}$

While slowing down, $\theta_2 = \bar{\omega} t = \frac{10.0\pi \text{ rad/s} + 0}{2} (12.0 \text{ s}) = 60.0\pi \text{ rad}$

So, $\theta_{\text{total}} = \theta_1 + \theta_2 = 100\pi \text{ rad} = \boxed{50.0 \text{ rev}}$

P10.8 $\theta_f - \theta_i = \omega_i t + \frac{1}{2} \alpha t^2$ and $\omega_f = \omega_i + \alpha t$ are two equations in two unknowns ω_i and α

$$\begin{aligned}\omega_i = \omega_f - \alpha t: \quad \theta_f - \theta_i &= (\omega_f - \alpha t)t + \frac{1}{2} \alpha t^2 = \omega_f t - \frac{1}{2} \alpha t^2 \\ 37.0 \text{ rev} \left(\frac{2\pi \text{ rad}}{1 \text{ rev}} \right) &= 98.0 \text{ rad/s} (3.00 \text{ s}) - \frac{1}{2} \alpha (3.00 \text{ s})^2 \\ 232 \text{ rad} = 294 \text{ rad} - (4.50 \text{ s}^2) \alpha: \quad \alpha &= \frac{61.5 \text{ rad}}{4.50 \text{ s}^2} = \boxed{13.7 \text{ rad/s}^2}\end{aligned}$$

P10.9 (a) $\omega = \frac{\Delta\theta}{\Delta t} = \frac{1 \text{ rev}}{1 \text{ day}} = \frac{2\pi \text{ rad}}{86\,400 \text{ s}} = \boxed{7.27 \times 10^{-5} \text{ rad/s}}$

(b) $\Delta t = \frac{\Delta\theta}{\omega} = \frac{107^\circ}{7.27 \times 10^{-5} \text{ rad/s}} \left(\frac{2\pi \text{ rad}}{360^\circ} \right) = \boxed{2.57 \times 10^4 \text{ s}}$ or 428 min

***P10.10** The location of the dog is described by $\theta_d = (0.750 \text{ rad/s})t$. For the bone,

$$\theta_b = \frac{1}{3} 2\pi \text{ rad} + \frac{1}{2} 0.015 \text{ rad/s}^2 t^2.$$

We look for a solution to

$$\begin{aligned}0.75t &= \frac{2\pi}{3} + 0.0075t^2 \\ 0 &= 0.0075t^2 - 0.75t + 2.09 = 0 \\ t &= \frac{0.75 \pm \sqrt{0.75^2 - 4(0.0075)(2.09)}}{0.015} = 2.88 \text{ s or } 97.1 \text{ s}\end{aligned}$$

The dog and bone will also pass if $0.75t = \frac{2\pi}{3} - 2\pi + 0.0075t^2$ or if $0.75t = \frac{2\pi}{3} + 2\pi + 0.0075t^2$ that is, if either the dog or the turntable gains a lap on the other. The first equation has

$$t = \frac{0.75 \pm \sqrt{0.75^2 - 4(0.0075)(-4.19)}}{0.015} = 105 \text{ s or } -5.30 \text{ s}$$

only one positive root representing a physical answer. The second equation has

$$t = \frac{0.75 \pm \sqrt{0.75^2 - 4(0.0075)(8.38)}}{0.015} = 12.8 \text{ s or } 87.2 \text{ s}.$$

In order, the dog passes the bone at $\boxed{2.88 \text{ s}}$ after the merry-go-round starts to turn, and again at $\boxed{12.8 \text{ s}}$ and 26.6 s, after gaining laps on the bone. The bone passes the dog at 73.4 s, 87.2 s, 97.1 s, 105 s, and so on, after the start.

Section 10.3 Angular and Linear Quantities

P10.11 Estimate the tire's radius at 0.250 m and miles driven as 10 000 per year.

$$\theta = \frac{s}{r} = \frac{1.00 \times 10^4 \text{ mi}}{0.250 \text{ m}} \left(\frac{1609 \text{ m}}{1 \text{ mi}} \right) = 6.44 \times 10^7 \text{ rad/yr}$$

$$\theta = 6.44 \times 10^7 \text{ rad/yr} \left(\frac{1 \text{ rev}}{2\pi \text{ rad}} \right) = 1.02 \times 10^7 \text{ rev/yr or } \boxed{\sim 10^7 \text{ rev/yr}}$$

P10.12 (a) $v = r\omega; \omega = \frac{v}{r} = \frac{45.0 \text{ m/s}}{250 \text{ m}} = \boxed{0.180 \text{ rad/s}}$

(b) $a_r = \frac{v^2}{r} = \frac{(45.0 \text{ m/s})^2}{250 \text{ m}} = \boxed{8.10 \text{ m/s}^2 \text{ toward the center of track}}$

P10.13 Given $r = 1.00 \text{ m}$, $\alpha = 4.00 \text{ rad/s}^2$, $\omega_i = 0$ and $\theta_i = 57.3^\circ = 1.00 \text{ rad}$

(a) $\omega_f = \omega_i + \alpha t = 0 + \alpha t$

At $t = 2.00 \text{ s}$, $\omega_f = 4.00 \text{ rad/s}^2 (2.00 \text{ s}) = \boxed{8.00 \text{ rad/s}}$

(b) $v = r\omega = 1.00 \text{ m}(8.00 \text{ rad/s}) = \boxed{8.00 \text{ m/s}}$

$|a_r| = a_c = r\omega^2 = 1.00 \text{ m}(8.00 \text{ rad/s})^2 = 64.0 \text{ m/s}^2$

$a_t = r\alpha = 1.00 \text{ m}(4.00 \text{ rad/s}^2) = 4.00 \text{ m/s}^2$

The magnitude of the total acceleration is:

$$a = \sqrt{a_r^2 + a_t^2} = \sqrt{(64.0 \text{ m/s}^2)^2 + (4.00 \text{ m/s}^2)^2} = \boxed{64.1 \text{ m/s}^2}$$

The direction of the total acceleration vector makes an angle ϕ with respect to the radius to point P :

$$\phi = \tan^{-1} \left(\frac{a_t}{a_c} \right) = \tan^{-1} \left(\frac{4.00}{64.0} \right) = \boxed{3.58^\circ}$$

(c) $\theta_f = \theta_i + \omega_i t + \frac{1}{2} \alpha t^2 = (1.00 \text{ rad}) + \frac{1}{2} (4.00 \text{ rad/s}^2) (2.00 \text{ s})^2 = \boxed{9.00 \text{ rad}}$

- *P10.14** (a) Consider a tooth on the front sprocket. It gives this speed, relative to the frame, to the link of the chain it engages:

$$v = r\omega = \left(\frac{0.152 \text{ m}}{2}\right) 76 \text{ rev/min} \left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right) \left(\frac{1 \text{ min}}{60 \text{ s}}\right) = \boxed{0.605 \text{ m/s}}$$

- (b) Consider the chain link engaging a tooth on the rear sprocket:

$$\omega = \frac{v}{r} = \frac{0.605 \text{ m/s}}{\left(\frac{0.07 \text{ m}}{2}\right)} = \boxed{17.3 \text{ rad/s}}$$

- (c) Consider the wheel tread and the road. A thread could be unwinding from the tire with this speed relative to the frame:

$$v = r\omega = \left(\frac{0.673 \text{ m}}{2}\right) 17.3 \text{ rad/s} = \boxed{5.82 \text{ m/s}}$$

- (d) We did not need to know the length of the pedal cranks, but we could use that information to find the linear speed of the pedals:

$$v = r\omega = 0.175 \text{ m} \cdot 7.96 \text{ rad/s} \left(\frac{1}{1 \text{ rad}}\right) = 1.39 \text{ m/s}$$

P10.15 (a) $\omega = \frac{v}{r} = \frac{25.0 \text{ m/s}}{1.00 \text{ m}} = \boxed{25.0 \text{ rad/s}}$

(b) $\omega_f^2 = \omega_i^2 + 2\alpha(\Delta\theta)$

$$\alpha = \frac{\omega_f^2 - \omega_i^2}{2(\Delta\theta)} = \frac{(25.0 \text{ rad/s})^2 - 0}{2[(1.25 \text{ rev})(2\pi \text{ rad/rev})]} = \boxed{39.8 \text{ rad/s}^2}$$

(c) $\Delta t = \frac{\Delta\omega}{\alpha} = \frac{25.0 \text{ rad/s}}{39.8 \text{ rad/s}^2} = \boxed{0.628 \text{ s}}$

P10.16 (a) $s = \bar{v}t = (11.0 \text{ m/s})(9.00 \text{ s}) = 99.0 \text{ m}$

$$\theta = \frac{s}{r} = \frac{99.0 \text{ m}}{0.290 \text{ m}} = 341 \text{ rad} = \boxed{54.3 \text{ rev}}$$

(b) $\omega_f = \frac{v_f}{r} = \frac{22.0 \text{ m/s}}{0.290 \text{ m}} = 75.9 \text{ rad/s} = \boxed{12.1 \text{ rev/s}}$

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P10.17 (a) $\omega = 2\pi f = \frac{2\pi \text{ rad}}{1 \text{ rev}} \left(\frac{1 \text{ 200 rev}}{60.0 \text{ s}} \right) = \boxed{126 \text{ rad/s}}$

(b) $v = \omega r = (126 \text{ rad/s})(3.00 \times 10^{-2} \text{ m}) = \boxed{3.77 \text{ m/s}}$

(c) $a_c = \omega^2 r = (126)^2 (8.00 \times 10^{-2}) = 1 \text{ 260 m/s}^2$ so $\mathbf{a}_r = \boxed{1.26 \text{ km/s}^2 \text{ toward the center}}$

(d) $s = r\theta = \omega r t = (126 \text{ rad/s})(8.00 \times 10^{-2} \text{ m})(2.00 \text{ s}) = \boxed{20.1 \text{ m}}$

P10.18 The force of static friction must act forward and then more and more inward on the tires, to produce both tangential and centripetal acceleration. Its tangential component is $m(1.70 \text{ m/s}^2)$. Its radially inward component is $\frac{mv^2}{r}$. This takes the maximum value

$$m\omega_f^2 r = mr(\omega_i^2 + 2\alpha\Delta\theta) = mr\left(0 + 2\alpha\frac{\pi}{2}\right) = m\pi r\alpha = m\pi a_t = m\pi(1.70 \text{ m/s}^2).$$

With skidding impending we have $\sum F_y = ma_y$, $+n - mg = 0$, $n = mg$

$$f_s = \mu_s n = \mu_s mg = \sqrt{m^2(1.70 \text{ m/s}^2)^2 + m^2\pi^2(1.70 \text{ m/s}^2)^2}$$

$$\mu_s = \frac{1.70 \text{ m/s}^2}{g} \sqrt{1 + \pi^2} = \boxed{0.572}$$

***P10.19** (a) Let R_E represent the radius of the Earth. The base of the building moves east at $v_1 = \omega R_E$ where ω is one revolution per day. The top of the building moves east at $v_2 = \omega(R_E + h)$. Its eastward speed relative to the ground is $v_2 - v_1 = \omega h$. The object's time of fall is given by $\Delta y = 0 + \frac{1}{2}gt^2$, $t = \sqrt{\frac{2h}{g}}$. During its fall the object's eastward motion is unimpeded so its

deflection distance is $\Delta x = (v_2 - v_1)t = \omega h \sqrt{\frac{2h}{g}} = \boxed{\omega h^{3/2} \left(\frac{2}{g}\right)^{1/2}}$.

(b) $\frac{2\pi \text{ rad}}{86 \text{ 400 s}} (50 \text{ m})^{3/2} \left(\frac{2 \text{ s}^2}{9.8 \text{ m}}\right)^{1/2} = \boxed{1.16 \text{ cm}}$

(c) The deflection is only 0.02% of the original height, so it is negligible in many practical cases.

Section 10.4 Rotational Energy

P10.20 $m_1 = 4.00 \text{ kg}$, $r_1 = |y_1| = 3.00 \text{ m}$;

$m_2 = 2.00 \text{ kg}$, $r_2 = |y_2| = 2.00 \text{ m}$;

$m_3 = 3.00 \text{ kg}$, $r_3 = |y_3| = 4.00 \text{ m}$;

$\omega = 2.00 \text{ rad/s}$ about the x -axis

(a) $I_x = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2$

$$I_x = 4.00(3.00)^2 + 2.00(2.00)^2 + 3.00(4.00)^2 = \boxed{92.0 \text{ kg} \cdot \text{m}^2}$$

$$K_R = \frac{1}{2} I_x \omega^2 = \frac{1}{2} (92.0) (2.00)^2 = \boxed{184 \text{ J}}$$

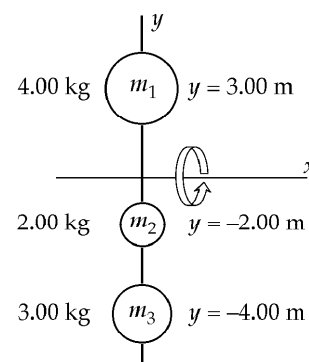


FIG. P10.20

(b) $v_1 = r_1 \omega = 3.00(2.00) = \boxed{6.00 \text{ m/s}}$

$v_2 = r_2 \omega = 2.00(2.00) = \boxed{4.00 \text{ m/s}}$

$v_3 = r_3 \omega = 4.00(2.00) = \boxed{8.00 \text{ m/s}}$

$$K_1 = \frac{1}{2} m_1 v_1^2 = \frac{1}{2} (4.00) (6.00)^2 = 72.0 \text{ J}$$

$$K_2 = \frac{1}{2} m_2 v_2^2 = \frac{1}{2} (2.00) (4.00)^2 = 16.0 \text{ J}$$

$$K_3 = \frac{1}{2} m_3 v_3^2 = \frac{1}{2} (3.00) (8.00)^2 = 96.0 \text{ J}$$

$$K = K_1 + K_2 + K_3 = 72.0 + 16.0 + 96.0 = \boxed{184 \text{ J}} = \frac{1}{2} I_x \omega^2$$

P10.21 (a) $I = \sum_j m_j r_j^2$

In this case,

$$r_1 = r_2 = r_3 = r_4$$

$$r = \sqrt{(3.00 \text{ m})^2 + (2.00 \text{ m})^2} = \sqrt{13.0} \text{ m}$$

$$I = [\sqrt{13.0} \text{ m}]^2 [3.00 + 2.00 + 2.00 + 4.00] \text{ kg}$$

$$= \boxed{143 \text{ kg} \cdot \text{m}^2}$$

(b) $K_R = \frac{1}{2} I \omega^2 = \frac{1}{2} (143 \text{ kg} \cdot \text{m}^2) (6.00 \text{ rad/s})^2$

$$= \boxed{2.57 \times 10^3 \text{ J}}$$

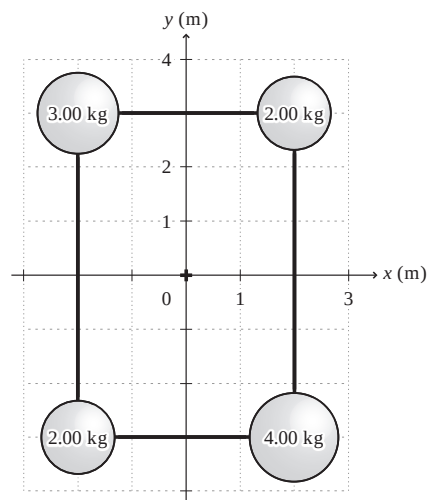


FIG. P10.21

P10.22 $I = Mx^2 + m(L-x)^2$

$$\frac{dI}{dx} = 2Mx - 2m(L-x) = 0 \quad (\text{for an extremum})$$

$$\therefore x = \frac{mL}{M+m}$$

$\frac{d^2I}{dx^2} = 2m + 2M$; therefore I is minimum when the axis of rotation passes through $x = \frac{mL}{M+m}$ which is also the center of mass of the system. The moment of inertia about an axis passing through x is

$$I_{\text{CM}} = M \left[\frac{mL}{M+m} \right]^2 + m \left[1 - \frac{m}{M+m} \right]^2 L^2 = \frac{Mm}{M+m} L^2 = \mu L^2$$

where $\mu = \frac{Mm}{M+m}$.

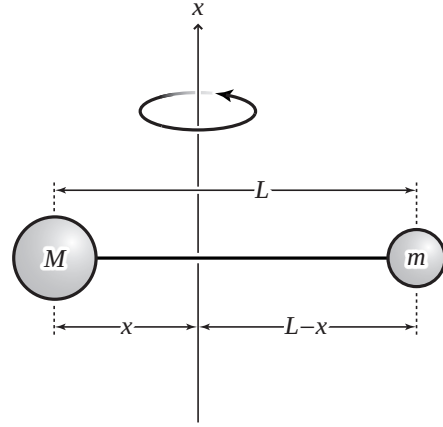


FIG. P10.22

Section 10.5 Calculation of Moments of Inertia

P10.23 We assume the rods are thin, with radius much less than L . Call the junction of the rods the origin of coordinates, and the axis of rotation the z -axis.

For the rod along the y -axis, $I = \frac{1}{3} mL^2$ from the table.

For the rod parallel to the z -axis, the parallel-axis theorem gives

$$I = \frac{1}{2} mr^2 + m \left(\frac{L}{2} \right)^2 \cong \frac{1}{4} mL^2$$

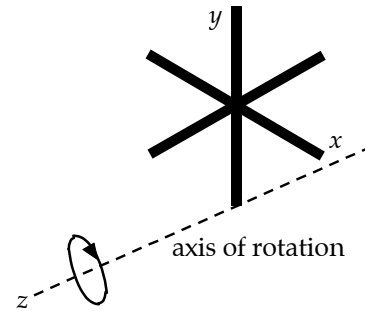


FIG. P10.23

In the rod along the x -axis, the bit of material between x and $x+dx$ has mass $\left(\frac{m}{L}\right)dx$ and is at

distance $r = \sqrt{x^2 + \left(\frac{L}{2}\right)^2}$ from the axis of rotation. The total rotational inertia is:

$$\begin{aligned} I_{\text{total}} &= \frac{1}{3} mL^2 + \frac{1}{4} mL^2 + \int_{-L/2}^{L/2} \left(x^2 + \frac{L^2}{4} \right) \left(\frac{m}{L} \right) dx \\ &= \frac{7}{12} mL^2 + \left(\frac{m}{L} \right) \frac{x^3}{3} \Big|_{-L/2}^{L/2} + \frac{mL}{4} x \Big|_{-L/2}^{L/2} \\ &= \frac{7}{12} mL^2 + \frac{mL^2}{12} + \frac{mL^2}{4} = \boxed{\frac{11mL^2}{12}} \end{aligned}$$

Note: The moment of inertia of the rod along the x axis can also be calculated from the parallel-axis theorem as $\frac{1}{12} mL^2 + m \left(\frac{L}{2} \right)^2$.

- P10.24** Treat the tire as consisting of three parts. The two sidewalls are each treated as a hollow cylinder of inner radius 16.5 cm, outer radius 30.5 cm, and height 0.635 cm. The tread region is treated as a hollow cylinder of inner radius 30.5 cm, outer radius 33.0 cm, and height 20.0 cm.

Use $I = \frac{1}{2}m(R_1^2 + R_2^2)$ for the moment of inertia of a hollow cylinder.

Sidewall:

$$m = \pi[(0.305 \text{ m})^2 - (0.165 \text{ m})^2](6.35 \times 10^{-3} \text{ m})(1.10 \times 10^3 \text{ kg/m}^3) = 1.44 \text{ kg}$$

$$I_{\text{side}} = \frac{1}{2}(1.44 \text{ kg})[(0.165 \text{ m})^2 + (0.305 \text{ m})^2] = 8.68 \times 10^{-2} \text{ kg} \cdot \text{m}^2$$

Tread:

$$m = \pi[(0.330 \text{ m})^2 - (0.305 \text{ m})^2](0.200 \text{ m})(1.10 \times 10^3 \text{ kg/m}^3) = 11.0 \text{ kg}$$

$$I_{\text{tread}} = \frac{1}{2}(11.0 \text{ kg})[(0.330 \text{ m})^2 + (0.305 \text{ m})^2] = 1.11 \text{ kg} \cdot \text{m}^2$$

Entire Tire:

$$I_{\text{total}} = 2I_{\text{side}} + I_{\text{tread}} = 2(8.68 \times 10^{-2} \text{ kg} \cdot \text{m}^2) + 1.11 \text{ kg} \cdot \text{m}^2 = \boxed{1.28 \text{ kg} \cdot \text{m}^2}$$

- P10.25** Every particle in the door could be slid straight down into a high-density rod across its bottom, without changing the particle's distance from the rotation axis of the door. Thus, a rod 0.870 m long with mass 23.0 kg, pivoted about one end, has the same rotational inertia as the door:

$$I = \frac{1}{3}ML^2 = \frac{1}{3}(23.0 \text{ kg})(0.870 \text{ m})^2 = \boxed{5.80 \text{ kg} \cdot \text{m}^2}.$$

The height of the door is unnecessary data.

- P10.26** Model your body as a cylinder of mass 60.0 kg and circumference 75.0 cm. Then its radius is

$$\frac{0.750 \text{ m}}{2\pi} = 0.120 \text{ m}$$

and its moment of inertia is

$$\frac{1}{2}MR^2 = \frac{1}{2}(60.0 \text{ kg})(0.120 \text{ m})^2 = 0.432 \text{ kg} \cdot \text{m}^2 \sim \boxed{10^0 \text{ kg} \cdot \text{m}^2 = 1 \text{ kg} \cdot \text{m}^2}.$$

P10.27 For a spherical shell $dI = \frac{2}{3} dmr^2 = \frac{2}{3} [4\pi r^2 dr] \rho r^2$

$$\begin{aligned}
 I &= \int dI = \int \frac{2}{3} (4\pi r^2) r^2 \rho(r) dr \\
 I &= \int_0^R \frac{2}{3} (4\pi r^4) \left(14.2 - 11.6 \frac{r}{R} \right) (10^3 \text{ kg/m}^3) dr \\
 &= \left(\frac{2}{3} \right) 4\pi (14.2 \times 10^3) \frac{R^5}{5} - \left(\frac{2}{3} \right) 4\pi (11.6 \times 10^3) \frac{R^5}{6} \\
 I &= \frac{8\pi}{3} (10^3) R^5 \left(\frac{14.2}{5} - \frac{11.6}{6} \right) \\
 M &= \int dm = \int_0^R 4\pi r^2 \left(14.2 - 11.6 \frac{r}{R} \right) 10^3 dr \\
 &= 4\pi \times 10^3 \left(\frac{14.2}{3} - \frac{11.6}{4} \right) R^3 \\
 \frac{I}{MR^2} &= \frac{(8\pi/3)(10^3)R^5(14.2/5 - 11.6/6)}{4\pi \times 10^3 R^3 R^2 (14.2/3 - 11.6/4)} = \frac{2}{3} \left(\frac{.907}{1.83} \right) = 0.330 \\
 \therefore I &= \boxed{0.330 MR^2}
 \end{aligned}$$

- *P10.28** (a) By similar triangles, $\frac{y}{x} = \frac{h}{L}$, $y = \frac{hx}{L}$. The area of the front face is $\frac{1}{2} hL$. The volume of the plate is $\frac{1}{2} hLw$. Its density is $\rho = \frac{M}{V} = \frac{M}{\frac{1}{2} hLw} = \frac{2M}{hLw}$. The mass of the ribbon is

$$dm = \rho dV = \rho y w dx = \frac{2My w dx}{hLw} = \frac{2Mhx}{hLL} dx = \frac{2Mx dx}{L^2}.$$

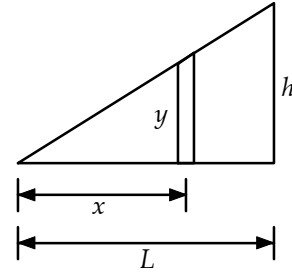


FIG. P10.28

The moment of inertia is

$$I = \int_{\text{all mass}} r^2 dm = \int_{x=0}^L x^2 \frac{2Mx dx}{L^2} = \frac{2M}{L^2} \int_0^L x^3 dx = \frac{2M}{L^2} \frac{L^4}{4} = \boxed{\frac{ML^2}{2}}.$$

- (b) From the parallel axis theorem $I = I_{\text{CM}} + M \left(\frac{2L}{3} \right)^2 = I_{\text{CM}} + \frac{4ML^2}{9}$ and

$I_h = I_{\text{CM}} + M \left(\frac{L}{3} \right)^2 = I_{\text{CM}} + \frac{ML^2}{9}$. The two triangles constitute a rectangle with moment of inertia $I_{\text{CM}} + \frac{4ML^2}{9} + I_{\text{CM}} + \frac{ML^2}{9} = \frac{1}{3} (2M)L^2$. Then $2I_{\text{CM}} = \frac{1}{9} ML^2$

$$I = I_{\text{CM}} + \frac{4ML^2}{9} = \frac{1}{18} ML^2 + \frac{8}{18} ML^2 = \boxed{\frac{1}{2} ML^2}.$$

- *P10.29** We consider the cam as the superposition of the original solid disk and a disk of negative mass cut from it. With half the radius, the cut-away part has one-quarter the face area and one-quarter the volume and one-quarter the mass M_0 of the original solid cylinder:

$$M_0 - \frac{1}{4}M_0 = M \quad M_0 = \frac{4}{3}M.$$

By the parallel-axis theorem, the original cylinder had moment of inertia

$$I_{\text{CM}} + M_0 \left(\frac{R}{2} \right)^2 = \frac{1}{2} M_0 R^2 + M_0 \frac{R^2}{4} = \frac{3}{4} M_0 R^2.$$

The negative-mass portion has $I = \frac{1}{2} \left(-\frac{1}{4} M_0 \right) \left(\frac{R}{2} \right)^2 = -\frac{M_0 R^2}{32}$. The whole cam has

$$I = \frac{3}{4} M_0 R^2 - \frac{M_0 R^2}{32} = \frac{23}{32} M_0 R^2 = \frac{23}{32} \frac{4}{3} M R^2 = \frac{23}{24} M R^2 \text{ and } K = \frac{1}{2} I \omega^2 = \frac{1}{2} \frac{23}{24} M R^2 \omega^2 = \boxed{\frac{23}{48} M R^2 \omega^2}.$$

Section 10.6 Torque

- P10.30** Resolve the 100 N force into components perpendicular to and parallel to the rod, as

$$F_{\text{par}} = (100 \text{ N}) \cos 57.0^\circ = 54.5 \text{ N}$$

$$\text{and } F_{\text{perp}} = (100 \text{ N}) \sin 57.0^\circ = 83.9 \text{ N}$$

The torque of F_{par} is zero since its line of action passes through the pivot point.

$$\text{The torque of } F_{\text{perp}} \text{ is } \tau = 83.9 \text{ N}(2.00 \text{ m}) = \boxed{168 \text{ N} \cdot \text{m}} \text{ (clockwise)}$$

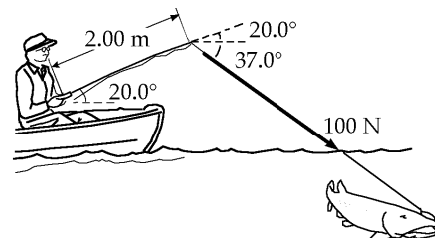


FIG. P10.30

- P10.31** $\sum \tau = 0.100 \text{ m}(12.0 \text{ N}) - 0.250 \text{ m}(9.00 \text{ N}) - 0.250 \text{ m}(10.0 \text{ N}) = \boxed{-3.55 \text{ N} \cdot \text{m}}$

The thirty-degree angle is unnecessary information.

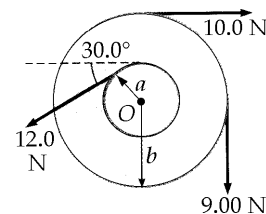


FIG. P10.31

- P10.32** The normal force exerted by the ground on each wheel is

$$n = \frac{mg}{4} = \frac{(1500 \text{ kg})(9.80 \text{ m/s}^2)}{4} = 3680 \text{ N}$$

The torque of friction can be as large as

$$\tau_{\text{max}} = f_{\text{max}} r = (\mu_s n) r = (0.800)(3680 \text{ N})(0.300 \text{ m}) = \boxed{882 \text{ N} \cdot \text{m}}$$

The torque of the axle on the wheel can be equally as large as the light wheel starts to turn without slipping.

- P10.33** In the previous problem we calculated the maximum torque that can be applied without skidding to be $882 \text{ N} \cdot \text{m}$. This same torque is to be applied by the frictional force, f , between the brake pad and the rotor for this wheel. Since the wheel is slipping against the brake pad, we use the coefficient of kinetic friction to calculate the normal force.

$$\tau = fr = (\mu_k n)r, \text{ so } n = \frac{\tau}{\mu_k r} = \frac{882 \text{ N} \cdot \text{m}}{(0.500)(0.220 \text{ m})} = 8.02 \times 10^3 \text{ N} = \boxed{8.02 \text{ kN}}$$

Section 10.7 Relationship Between Torque and Angular Acceleration

P10.34 (a) $I = \frac{1}{2}MR^2 = \frac{1}{2}(2.00 \text{ kg})(7.00 \times 10^{-2} \text{ m})^2 = 4.90 \times 10^{-3} \text{ kg} \cdot \text{m}^2$
 $\alpha = \frac{\tau}{I} = \frac{0.600}{4.90 \times 10^{-3}} = 122 \text{ rad/s}^2$
 $\alpha = \frac{\Delta\omega}{\Delta t}$
 $\Delta t = \frac{\Delta\omega}{\alpha} = \frac{1200\left(\frac{2\pi}{60}\right)}{122} = \boxed{1.03 \text{ s}}$

(b) $\Delta\theta = \frac{1}{2}\alpha t^2 = \frac{1}{2}(122 \text{ rad/s}^2)(1.03 \text{ s})^2 = 64.7 \text{ rad} = \boxed{10.3 \text{ rev}}$

P10.35 $m = 0.750 \text{ kg}, F = 0.800 \text{ N}$

(a) $\tau = rF = 30.0 \text{ m}(0.800 \text{ N}) = \boxed{24.0 \text{ N} \cdot \text{m}}$

(b) $\alpha = \frac{\tau}{I} = \frac{rF}{mr^2} = \frac{24.0}{0.750(30.0)^2} = \boxed{0.0356 \text{ rad/s}^2}$

(c) $a_t = \alpha r = 0.0356(30.0) = \boxed{1.07 \text{ m/s}^2}$

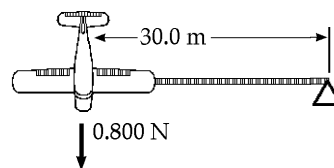


FIG. P10.35

P10.36 $\omega_f = \omega_i + \alpha t:$ $10.0 \text{ rad/s} = 0 + \alpha(6.00 \text{ s})$
 $\alpha = \frac{10.00}{6.00} \text{ rad/s}^2 = 1.67 \text{ rad/s}^2$

(a) $\sum \tau = 36.0 \text{ N} \cdot \text{m} = I\alpha:$ $I = \frac{\sum \tau}{\alpha} = \frac{36.0 \text{ N} \cdot \text{m}}{1.67 \text{ rad/s}^2} = \boxed{21.6 \text{ kg} \cdot \text{m}^2}$

(b) $\omega_f = \omega_i + \alpha t:$ $0 = 10.0 + \alpha(60.0)$
 $\alpha = -0.167 \text{ rad/s}^2$
 $|\tau| = |I\alpha| = (21.6 \text{ kg} \cdot \text{m}^2)(0.167 \text{ rad/s}^2) = \boxed{3.60 \text{ N} \cdot \text{m}}$

(c) Number of revolutions $\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$
 During first 6.00 s $\theta_f = \frac{1}{2}(1.67)(6.00)^2 = 30.1 \text{ rad}$
 During next 60.0 s $\theta_f = 10.0(60.0) - \frac{1}{2}(0.167)(60.0)^2 = 299 \text{ rad}$
 $\theta_{\text{total}} = 329 \text{ rad} \left(\frac{1 \text{ rev}}{2\pi \text{ rad}} \right) = \boxed{52.4 \text{ rev}}$

P10.37For m_1 ,

$$\begin{aligned}\sum F_y = ma_y: & +n - m_1g = 0 \\ n_1 &= m_1g = 19.6 \text{ N} \\ f_{k1} &= \mu_k n_1 = 7.06 \text{ N}\end{aligned}$$

$$\sum F_x = ma_x: -7.06 \text{ N} + T_1 = (2.00 \text{ kg})a \quad (1)$$

For the pulley,

$$\begin{aligned}\sum \tau = I\alpha: & -T_1R + T_2R = \frac{1}{2}MR^2\left(\frac{a}{R}\right) \\ -T_1 + T_2 &= \frac{1}{2}(10.0 \text{ kg})a \\ -T_1 + T_2 &= (5.00 \text{ kg})a\end{aligned}$$

For m_2 ,

$$\begin{aligned}+n_2 - m_2g \cos \theta &= 0 \\ n_2 &= 6.00 \text{ kg}(9.80 \text{ m/s}^2)(\cos 30.0^\circ) \\ &= 50.9 \text{ N}\end{aligned}$$

$$f_{k2} = \mu_k n_2$$

$$\begin{aligned}= 18.3 \text{ N}: & -18.3 \text{ N} - T_2 + m_2 \sin \theta = m_2a \\ -18.3 \text{ N} - T_2 + 29.4 \text{ N} &= (6.00 \text{ kg})a \quad (3)\end{aligned}$$

(a) Add equations (1), (2), and (3):

$$\begin{aligned}-7.06 \text{ N} - 18.3 \text{ N} + 29.4 \text{ N} &= (13.0 \text{ kg})a \\ a &= \frac{4.01 \text{ N}}{13.0 \text{ kg}} = \boxed{0.309 \text{ m/s}^2}\end{aligned}$$

$$(b) \quad T_1 = 2.00 \text{ kg}(0.309 \text{ m/s}^2) + 7.06 \text{ N} = \boxed{7.67 \text{ N}}$$

$$T_2 = 7.67 \text{ N} + 5.00 \text{ kg}(0.309 \text{ m/s}^2) = \boxed{9.22 \text{ N}}$$

P10.38

$$I = \frac{1}{2}mR^2 = \frac{1}{2}(100 \text{ kg})(0.500 \text{ m})^2 = 12.5 \text{ kg} \cdot \text{m}^2$$

$$\omega_i = 50.0 \text{ rev/min} = 5.24 \text{ rad/s}$$

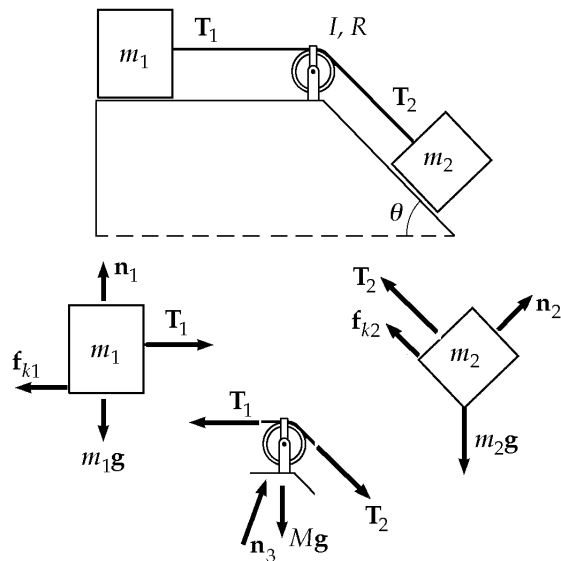
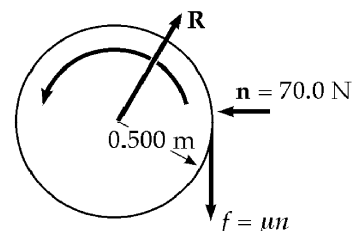
$$\alpha = \frac{\omega_f - \omega_i}{t} = \frac{0 - 5.24 \text{ rad/s}}{6.00 \text{ s}} = -0.873 \text{ rad/s}^2$$

$$\tau = I\alpha = 12.5 \text{ kg} \cdot \text{m}^2(-0.873 \text{ rad/s}^2) = -10.9 \text{ N} \cdot \text{m}$$

The magnitude of the torque is given by $fR = 10.9 \text{ N} \cdot \text{m}$, where f is the force of friction.

$$\text{Therefore, } f = \frac{10.9 \text{ N} \cdot \text{m}}{0.500 \text{ m}} \quad \text{and} \quad f = \mu_k n$$

$$\text{yields } \mu_k = \frac{f}{n} = \frac{21.8 \text{ N}}{70.0 \text{ N}} = \boxed{0.312}$$

**FIG. P10.37****FIG. P10.38**

$$\begin{aligned}
 *P10.39 \quad \sum \tau &= I\alpha = \frac{1}{2}MR^2\alpha \\
 -135 \text{ N}(0.230 \text{ m}) + T(0.230 \text{ m}) &= \frac{1}{2}(80 \text{ kg})\left(\frac{1.25}{2} \text{ m}\right)^2(-1.67 \text{ rad/s}^2) \\
 T &= \boxed{21.5 \text{ N}}
 \end{aligned}$$

Section 10.8 Work, Power, and Energy in Rotational Motion

P10.40 The moment of inertia of a thin rod about an axis through one end is $I = \frac{1}{3}ML^2$. The total rotational kinetic energy is given as

$$K_R = \frac{1}{2}I_h\omega_h^2 + \frac{1}{2}I_m\omega_m^2$$

$$\text{with } I_h = \frac{m_h L_h^2}{3} = \frac{60.0 \text{ kg}(2.70 \text{ m})^2}{3} = 146 \text{ kg} \cdot \text{m}^2$$

$$\text{and } I_m = \frac{m_m L_m^2}{3} = \frac{100 \text{ kg}(4.50 \text{ m})^2}{3} = 675 \text{ kg} \cdot \text{m}^2$$

$$\text{In addition, } \omega_h = \frac{2\pi \text{ rad}}{12 \text{ h}} \left(\frac{1 \text{ h}}{3600 \text{ s}} \right) = 1.45 \times 10^{-4} \text{ rad/s}$$

$$\text{while } \omega_m = \frac{2\pi \text{ rad}}{1 \text{ h}} \left(\frac{1 \text{ h}}{3600 \text{ s}} \right) = 1.75 \times 10^{-3} \text{ rad/s}$$

$$\text{Therefore, } K_R = \frac{1}{2}(146)(1.45 \times 10^{-4})^2 + \frac{1}{2}(675)(1.75 \times 10^{-3})^2 = \boxed{1.04 \times 10^{-3} \text{ J}}$$

***P10.41** The power output of the bus is $\mathcal{P} = \frac{E}{\Delta t}$ where $E = \frac{1}{2}I\omega^2 = \frac{1}{2}\frac{1}{2}MR^2\omega^2$ is the stored energy and $\Delta t = \frac{\Delta x}{v}$ is the time it can roll. Then $\frac{1}{4}MR^2\omega^2 = \mathcal{P}\Delta t = \frac{\mathcal{P}\Delta x}{v}$ and

$$\Delta x = \frac{MR^2\omega^2 v}{4\mathcal{P}} = \frac{1600 \text{ kg}(0.65 \text{ m})^2(4000 \cdot \frac{2\pi}{60 \text{ s}})^2 11.1 \text{ m/s}}{4(18746 \text{ W})} = \boxed{24.5 \text{ km}}$$

P10.42 Work done $= F\Delta r = (5.57 \text{ N})(0.800 \text{ m}) = 4.46 \text{ J}$

$$\text{and Work} = \Delta K = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2$$

(The last term is zero because the top starts from rest.)

$$\text{Thus, } 4.46 \text{ J} = \frac{1}{2}(4.00 \times 10^{-4} \text{ kg} \cdot \text{m}^2)\omega_f^2$$

$$\text{and from this, } \omega_f = \boxed{149 \text{ rad/s}}$$

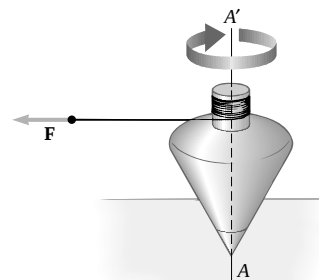


FIG. P10.42

***P10.43** (a)
$$I = \frac{1}{2}M(R_1^2 + R_2^2) = \frac{1}{2}(0.35 \text{ kg})[(0.02 \text{ m})^2 + (0.03 \text{ m})^2] = 2.28 \times 10^{-4} \text{ kg} \cdot \text{m}^2$$

$$(K_1 + K_2 + K_{\text{rot}} + U_{g2})_i - f_k \Delta x = (K_1 + K_2 + K_{\text{rot}})_f$$

$$\frac{1}{2}(0.850 \text{ kg})(0.82 \text{ m/s})^2 + \frac{1}{2}(0.42 \text{ kg})(0.82 \text{ m/s})^2 + \frac{1}{2}(2.28 \times 10^{-4} \text{ kg} \cdot \text{m}^2)\left(\frac{0.82 \text{ m/s}}{0.03 \text{ m}}\right)^2$$

$$+ 0.42 \text{ kg}(9.8 \text{ m/s}^2)(0.7 \text{ m}) - 0.25(0.85 \text{ kg})(9.8 \text{ m/s}^2)(0.7 \text{ m})$$

$$= \frac{1}{2}(0.85 \text{ kg})v_f^2 + \frac{1}{2}(0.42 \text{ kg})v_f^2 + \frac{1}{2}(2.28 \times 10^{-4} \text{ kg} \cdot \text{m}^2)\left(\frac{v_f}{0.03 \text{ m}}\right)^2$$

$$0.512 \text{ J} + 2.88 \text{ J} - 1.46 \text{ J} = (0.761 \text{ kg})v_f^2$$

$$v_f = \sqrt{\frac{1.94 \text{ J}}{0.761 \text{ kg}}} = \boxed{1.59 \text{ m/s}}$$

(b)
$$\omega = \frac{v}{r} = \frac{1.59 \text{ m/s}}{0.03 \text{ m}} = \boxed{53.1 \text{ rad/s}}$$

P10.44 We assume the rod is thin. For the compound object

$$I = \frac{1}{3}M_{\text{rod}}L^2 + \left[\frac{2}{5}m_{\text{ball}}R^2 + M_{\text{ball}}D^2 \right]$$

$$I = \frac{1}{3}1.20 \text{ kg}(0.240 \text{ m})^2 + \frac{2}{5}2.00 \text{ kg}(4.00 \times 10^{-2} \text{ m})^2 + 2.00 \text{ kg}(0.280 \text{ m})^2$$

$$I = 0.181 \text{ kg} \cdot \text{m}^2$$

(a)
$$K_f + U_f = K_i + U_i + \Delta E$$

$$\frac{1}{2}I\omega^2 + 0 = 0 + M_{\text{rod}}g\left(\frac{L}{2}\right) + M_{\text{ball}}g(L + R) + 0$$

$$\frac{1}{2}(0.181 \text{ kg} \cdot \text{m}^2)\omega^2 = 1.20 \text{ kg}(9.80 \text{ m/s}^2)(0.120 \text{ m}) + 2.00 \text{ kg}(9.80 \text{ m/s}^2)(0.280 \text{ m})$$

$$\frac{1}{2}(0.181 \text{ kg} \cdot \text{m}^2)\omega^2 = \boxed{6.90 \text{ J}}$$

(b)
$$\omega = \boxed{8.73 \text{ rad/s}}$$

(c)
$$v = r\omega = (0.280 \text{ m})8.73 \text{ rad/s} = \boxed{2.44 \text{ m/s}}$$

(d)
$$v_f^2 = v_i^2 + 2a(y_f - y_i)$$

$$v_f = \sqrt{0 + 2(9.80 \text{ m/s}^2)(0.280 \text{ m})} = 2.34 \text{ m/s}$$

The speed it attains in swinging is greater by $\frac{2.44}{2.34} = \boxed{1.043 \text{ 2 times}}$

302 Rotation of a Rigid Object About a Fixed Axis

P10.45 (a) For the counterweight,

$$\sum F_y = ma_y \text{ becomes: } 50.0 - T = \left(\frac{50.0}{9.80} \right) a$$

$$\text{For the reel } \sum \tau = I\alpha \text{ reads } TR = I\alpha = I \frac{a}{R}$$

$$\text{where } I = \frac{1}{2} MR^2 = 0.0938 \text{ kg} \cdot \text{m}^2$$

We substitute to eliminate the acceleration:

$$50.0 - T = 5.10 \left(\frac{TR^2}{I} \right)$$

$$T = \boxed{11.4 \text{ N}} \quad \text{and}$$

$$a = \frac{50.0 - 11.4}{5.10} = \boxed{7.57 \text{ m/s}^2}$$

$$v_f^2 = v_i^2 + 2a(x_f - x_i): \quad v_f = \sqrt{2(7.57)(6.00)} = \boxed{9.53 \text{ m/s}}$$

(b) Use conservation of energy for the system of the object, the reel, and the Earth:

$$\begin{aligned} (K + U)_i &= (K + U)_f: & mgh &= \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2 \\ 2mgh &= mv^2 + I \left(\frac{v^2}{R^2} \right) & &= v^2 \left(m + \frac{I}{R^2} \right) \\ v &= \sqrt{\frac{2mgh}{m + \frac{I}{R^2}}} = \sqrt{\frac{2(50.0 \text{ N})(6.00 \text{ m})}{5.10 \text{ kg} + \frac{0.0938}{(0.250)^2}}} = \boxed{9.53 \text{ m/s}} \end{aligned}$$

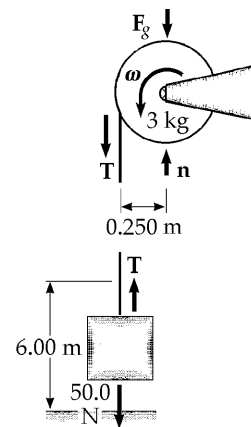


FIG. P10.45

P10.46 Choose the zero gravitational potential energy at the level where the masses pass.

$$\begin{aligned} K_f + U_{gf} &= K_i + U_{gi} + \Delta E \\ \frac{1}{2} m_1 v^2 + \frac{1}{2} m_2 v^2 + \frac{1}{2} I\omega^2 &= 0 + m_1 gh_{1i} + m_2 gh_{2i} + 0 \\ \frac{1}{2} (15.0 + 10.0) v^2 + \frac{1}{2} \left[\frac{1}{2} (3.00) R^2 \right] \left(\frac{v}{R} \right)^2 &= 15.0(9.80)(1.50) + 10.0(9.80)(-1.50) \\ \frac{1}{2} (26.5 \text{ kg}) v^2 &= 73.5 \text{ J} \Rightarrow v = \boxed{2.36 \text{ m/s}} \end{aligned}$$

P10.47 From conservation of energy for the object-turntable-cylinder-Earth system,

$$\begin{aligned} \frac{1}{2} I \left(\frac{v}{r} \right)^2 + \frac{1}{2} mv^2 &= mgh \\ I \frac{v^2}{r^2} &= 2mgh - mv^2 \\ I &= \boxed{mr^2 \left(\frac{2gh}{v^2} - 1 \right)} \end{aligned}$$

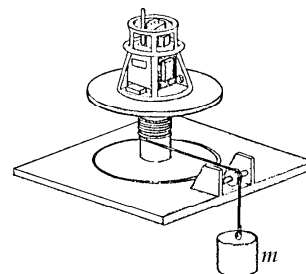


FIG. P10.47

P10.48 The moment of inertia of the cylinder is

$$I = \frac{1}{2}mr^2 = \frac{1}{2}(81.6 \text{ kg})(1.50 \text{ m})^2 = 91.8 \text{ kg} \cdot \text{m}^2$$

and the angular acceleration of the merry-go-round is found as

$$\alpha = \frac{\tau}{I} = \frac{(Fr)}{I} = \frac{(50.0 \text{ N})(1.50 \text{ m})}{(91.8 \text{ kg} \cdot \text{m}^2)} = 0.817 \text{ rad/s}^2.$$

At $t = 3.00 \text{ s}$, we find the angular velocity

$$\omega = \omega_i + \alpha t$$

$$\omega = 0 + (0.817 \text{ rad/s}^2)(3.00 \text{ s}) = 2.45 \text{ rad/s}$$

$$\text{and } K = \frac{1}{2}I\omega^2 = \frac{1}{2}(91.8 \text{ kg} \cdot \text{m}^2)(2.45 \text{ rad/s})^2 = \boxed{276 \text{ J}}.$$

P10.49 (a) Find the velocity of the CM

$$(K + U)_i = (K + U)_f$$

$$0 + mgR = \frac{1}{2}I\omega^2$$

$$\omega = \sqrt{\frac{2mgR}{I}} = \sqrt{\frac{2mgR}{\frac{3}{2}mR^2}}$$

$$v_{\text{CM}} = R\omega = R\sqrt{\frac{4g}{3R}} = \boxed{2\sqrt{\frac{Rg}{3}}}$$

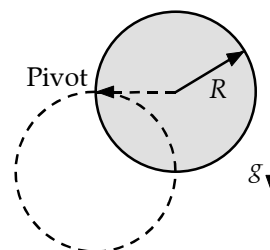


FIG. P10.49

$$(b) \quad v_L = 2v_{\text{CM}} = \boxed{4\sqrt{\frac{Rg}{3}}}$$

$$(c) \quad v_{\text{CM}} = \sqrt{\frac{2mgR}{2m}} = \boxed{\sqrt{Rg}}$$

***P10.50** (a) The moment of inertia of the cord on the spool is

$$\frac{1}{2}M(R_1^2 + R_2^2) = \frac{1}{2}0.1 \text{ kg}((0.015 \text{ m})^2 + (0.09 \text{ m})^2) = 4.16 \times 10^{-4} \text{ kg} \cdot \text{m}^2.$$

The protruding strand has mass $(10^{-2} \text{ kg/m})(0.16 \text{ m}) = 1.6 \times 10^{-3} \text{ kg}$ and

$$\begin{aligned} I &= I_{\text{CM}} + Md^2 = \frac{1}{12}ML^2 + Md^2 = 1.6 \times 10^{-3} \text{ kg} \left(\frac{1}{12}(0.16 \text{ m})^2 + (0.09 \text{ m} + 0.08 \text{ m})^2 \right) \\ &= 4.97 \times 10^{-5} \text{ kg} \cdot \text{m}^2 \end{aligned}$$

For the whole cord, $I = 4.66 \times 10^{-4} \text{ kg} \cdot \text{m}^2$. In speeding up, the average power is

$$\mathcal{P} = \frac{E}{\Delta t} = \frac{\frac{1}{2}I\omega^2}{\Delta t} = \frac{4.66 \times 10^{-4} \text{ kg} \cdot \text{m}^2 \left(\frac{2500 \cdot 2\pi}{60 \text{ s}} \right)^2}{2(0.215 \text{ s})} = \boxed{74.3 \text{ W}}$$

$$(b) \quad \mathcal{P} = \tau\omega = (7.65 \text{ N})(0.16 \text{ m} + 0.09 \text{ m}) \left(\frac{2000 \cdot 2\pi}{60 \text{ s}} \right) = \boxed{401 \text{ W}}$$

Section 10.9 Rolling Motion of a Rigid Object

P10.51 (a) $K_{\text{trans}} = \frac{1}{2}mv^2 = \frac{1}{2}(10.0 \text{ kg})(10.0 \text{ m/s})^2 = \boxed{500 \text{ J}}$

(b) $K_{\text{rot}} = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v^2}{r^2}\right) = \frac{1}{4}(10.0 \text{ kg})(10.0 \text{ m/s})^2 = \boxed{250 \text{ J}}$

(c) $K_{\text{total}} = K_{\text{trans}} + K_{\text{rot}} = \boxed{750 \text{ J}}$

P10.52 $W = K_f - K_i = (K_{\text{trans}} + K_{\text{rot}})_f - (K_{\text{trans}} + K_{\text{rot}})_i$

$$W = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 - 0 - 0 = \frac{1}{2}Mv^2 + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{v}{R}\right)^2$$

or $W = \boxed{\left(\frac{7}{10}\right)Mv^2}$

P10.53 (a) $\tau = I\alpha$
 $mgR \sin \theta = (I_{\text{CM}} + mR^2)\alpha$

$$a = \frac{mgR^2 \sin \theta}{I_{\text{CM}} + mR^2}$$

$$a_{\text{hoop}} = \frac{mgR^2 \sin \theta}{2mR^2} = \boxed{\frac{1}{2}g \sin \theta}$$

$$a_{\text{disk}} = \frac{mgR^2 \sin \theta}{\frac{3}{2}mR^2} = \boxed{\frac{2}{3}g \sin \theta}$$

The disk moves with $\frac{4}{3}$ the acceleration of the hoop.

(b) $Rf = I\alpha$

$$f = \mu n = \mu mg \cos \theta$$

$$\mu = \frac{f}{mg \cos \theta} = \frac{\frac{I\alpha}{R}}{mg \cos \theta} = \frac{\left(\frac{2}{3}g \sin \theta\right)\left(\frac{1}{2}mR^2\right)}{R^2 mg \cos \theta} = \boxed{\frac{1}{3} \tan \theta}$$

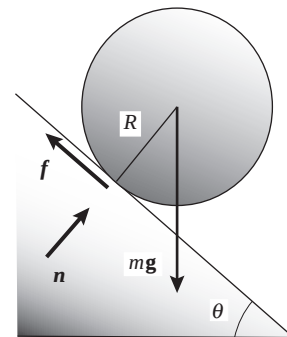


FIG. P10.53

P10.54 $K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}\left[m + \frac{I}{R^2}\right]v^2$ where $\omega = \frac{v}{R}$ since no slipping.

Also, $U_i = mgh$, $U_f = 0$, and $v_i = 0$

Therefore,

$$\frac{1}{2}\left[m + \frac{I}{R^2}\right]v^2 = mgh$$

Thus,

$$v^2 = \frac{2gh}{\left[1 + \left(\frac{I}{mR^2}\right)\right]}$$

For a disk,

$$I = \frac{1}{2}mR^2$$

So $v^2 = \frac{2gh}{1 + \frac{1}{2}}$ or

$$v_{\text{disk}} = \sqrt{\frac{4gh}{3}}$$

For a ring, $I = mR^2$ so $v^2 = \frac{2gh}{2}$ or

$$v_{\text{ring}} = \sqrt{gh}$$

Since $v_{\text{disk}} > v_{\text{ring}}$, **the disk** reaches the bottom first.

$$\text{P10.55} \quad \bar{v} = \frac{\Delta x}{\Delta t} = \frac{3.00 \text{ m}}{1.50 \text{ s}} = 2.00 \text{ m/s} = \frac{1}{2}(0 + v_f)$$

$$v_f = 4.00 \text{ m/s and } \omega_f = \frac{v_f}{r} = \frac{4.00 \text{ m/s}}{(6.38 \times 10^{-2} \text{ m})/2} = \frac{8.00}{6.38 \times 10^{-2}} \text{ rad/s}$$

We ignore internal friction and suppose the can rolls without slipping.

$$(K_{\text{trans}} + K_{\text{rot}} + U_g)_i + \Delta E_{\text{mech}} = (K_{\text{trans}} + K_{\text{rot}} + U_g)_f$$

$$(0 + 0 + mgy_i) + 0 = \left(\frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2 + 0 \right)$$

$$0.215 \text{ kg}(9.80 \text{ m/s}^2)[(3.00 \text{ m})\sin 25.0^\circ] = \frac{1}{2}(0.215 \text{ kg})(4.00 \text{ m/s})^2 + \frac{1}{2}I\left(\frac{8.00}{6.38 \times 10^{-2}} \text{ rad/s}\right)^2$$

$$2.67 \text{ J} = 1.72 \text{ J} + (7860 \text{ s}^{-2})I$$

$$I = \frac{0.951 \text{ kg} \cdot \text{m}^2/\text{s}^2}{7860 \text{ s}^{-2}} = \boxed{1.21 \times 10^{-4} \text{ kg} \cdot \text{m}^2}$$

The height of the can is unnecessary data.

- P10.56** (a) Energy conservation for the system of the ball and the Earth between the horizontal section and top of loop:

$$\frac{1}{2}mv_2^2 + \frac{1}{2}I\omega_2^2 + mgy_2 = \frac{1}{2}mv_1^2 + \frac{1}{2}I\omega_1^2$$

$$\frac{1}{2}mv_2^2 + \frac{1}{2}\left(\frac{2}{3}mr^2\right)\left(\frac{v_2}{r}\right)^2 + mgy_2$$

$$= \frac{1}{2}mv_1^2 + \frac{1}{2}\left(\frac{2}{3}mr^2\right)\left(\frac{v_1}{r}\right)^2$$

$$\frac{5}{6}v_2^2 + gy_2 = \frac{5}{6}v_1^2$$

$$v_2 = \sqrt{v_1^2 - \frac{6}{5}gy_2} = \sqrt{(4.03 \text{ m/s})^2 - \frac{6}{5}(9.80 \text{ m/s}^2)(0.900 \text{ m})} = \boxed{2.38 \text{ m/s}}$$

$$\text{The centripetal acceleration is } \frac{v_2^2}{r} = \frac{(2.38 \text{ m/s})^2}{0.450 \text{ m}} = 12.6 \text{ m/s}^2 > g$$

Thus, the ball must be in contact with the track, with the track pushing downward on it.

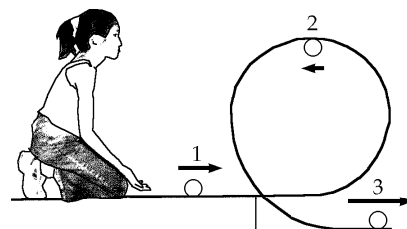


FIG. P10.56

$$(b) \quad \frac{1}{2}mv_3^2 + \frac{1}{2}\left(\frac{2}{3}mr^2\right)\left(\frac{v_3}{r}\right)^2 + mgy_3 = \frac{1}{2}mv_1^2 + \frac{1}{2}\left(\frac{2}{3}mr^2\right)\left(\frac{v_1}{r}\right)^2$$

$$v_3 = \sqrt{v_1^2 - \frac{6}{5}gy_3} = \sqrt{(4.03 \text{ m/s})^2 - \frac{6}{5}(9.80 \text{ m/s}^2)(-0.200 \text{ m})} = \boxed{4.31 \text{ m/s}}$$

$$(c) \quad \frac{1}{2}mv_2^2 + mgy_2 = \frac{1}{2}mv_1^2$$

$$v_2 = \sqrt{v_1^2 - 2gy_2} = \sqrt{(4.03 \text{ m/s})^2 - 2(9.80 \text{ m/s}^2)(0.900 \text{ m})} = \sqrt{-1.40 \text{ m}^2/\text{s}^2}$$

This result is imaginary. In the case where the ball does not roll, the ball starts with less energy than in part (a) and never makes it to the top of the loop.

Additional Problems

P10.57 $mg \frac{\ell}{2} \sin \theta = \frac{1}{3} m \ell^2 \alpha$

$$\alpha = \frac{3}{2} \frac{g}{\ell} \sin \theta$$

$$a_t = \left(\frac{3}{2} \frac{g}{\ell} \sin \theta \right) r$$

Then $\left(\frac{3}{2} \frac{g}{\ell} \right) r > g \sin \theta$

for $r > \frac{2}{3} \ell$

$\therefore$ About $\boxed{\frac{1}{3}}$ the length of the chimney will have a

tangential acceleration greater than $g \sin \theta$.

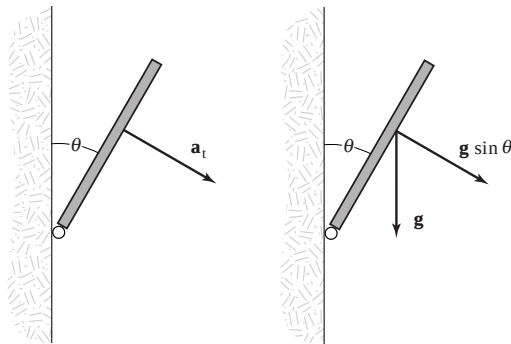


FIG. P10.57

P10.58 The resistive force on each ball is $R = D\rho A v^2$. Here $v = r\omega$, where r is the radius of each ball's path. The resistive torque on each ball is $\tau = rR$, so the total resistive torque on the three ball system is $\tau_{\text{total}} = 3rR$.

The power required to maintain a constant rotation rate is $\mathcal{P} = \tau_{\text{total}}\omega = 3rR\omega$. This required power may be written as

$$\mathcal{P} = \tau_{\text{total}}\omega = 3r[D\rho A(r\omega)^2]\omega = (3r^3 DA\omega^3)\rho$$

With $\omega = \frac{2\pi \text{ rad}}{1 \text{ rev}} \left(\frac{10^3 \text{ rev}}{1 \text{ min}} \right) \left(\frac{1 \text{ min}}{60.0 \text{ s}} \right) = \frac{1000\pi}{30.0} \text{ rad/s}$

$$\mathcal{P} = 3(0.100 \text{ m})^3 (0.600) (4.00 \times 10^{-4} \text{ m}^2) \left(\frac{1000\pi}{30.0 \text{ s}} \right)^3 \rho$$

or $\mathcal{P} = (0.827 \text{ m}^5/\text{s}^3)\rho$, where ρ is the density of the resisting medium.

(a) In air, $\rho = 1.20 \text{ kg/m}^3$,

and $\mathcal{P} = 0.827 \text{ m}^5/\text{s}^3 (1.20 \text{ kg/m}^3) = 0.992 \text{ N} \cdot \text{m/s} = \boxed{0.992 \text{ W}}$

(b) In water, $\rho = 1000 \text{ kg/m}^3$ and $\mathcal{P} = \boxed{827 \text{ W}}$.

P10.59 (a) $W = \Delta K = \frac{1}{2} I \omega_f^2 - \frac{1}{2} I \omega_i^2 = \frac{1}{2} I (\omega_f^2 - \omega_i^2)$ where $I = \frac{1}{2} m R^2$

$$= \left(\frac{1}{2} \right) \left(\frac{1}{2} \right) (1.00 \text{ kg}) (0.500 \text{ m})^2 [(8.00 \text{ rad/s})^2 - 0] = \boxed{4.00 \text{ J}}$$

(b) $t = \frac{\omega_f - 0}{\alpha} = \frac{\omega r}{a} = \frac{(8.00 \text{ rad/s})(0.500 \text{ m})}{2.50 \text{ m/s}^2} = \boxed{1.60 \text{ s}}$

(c) $\theta_f = \theta_i + \omega_i t + \frac{1}{2} \alpha t^2$; $\theta_i = 0$; $\omega_i = 0$

$$\theta_f = \frac{1}{2} \alpha t^2 = \frac{1}{2} \left(\frac{2.50 \text{ m/s}^2}{0.500 \text{ m}} \right) (1.60 \text{ s})^2 = 6.40 \text{ rad}$$

$$s = r\theta = (0.500 \text{ m})(6.40 \text{ rad}) = \boxed{3.20 \text{ m} < 4.00 \text{ m Yes}}$$

- *P10.60** The quantity of tape is constant. Then the area of the rings you see it fill is constant. This is expressed by $\pi r_t^2 - \pi r_s^2 = \pi r^2 - \pi r_s^2 + \pi r_2^2 - \pi r_s^2$ or $r_2 = \sqrt{r_t^2 + r_s^2 - r^2}$ is the outer radius of spool 2.

- (a) Where the tape comes off spool 1, $\omega_1 = \frac{v}{r}$. Where the tape joins spool 2, $\omega_2 = \frac{v}{r_2} = v(r_s^2 + r_t^2 - r^2)^{-1/2}$.
- (b) At the start, $r = r_t$ and $r_2 = r_s$ so $\omega_1 = \frac{v}{r_t}$ and $\omega_2 = \frac{v}{r_s}$. The takeup reel must spin at maximum speed. At the end, $r = r_s$ and $r_2 = r_t$ so $\omega_2 = \frac{v}{r_t}$ and $\omega_1 = \frac{v}{r_s}$. The angular speeds are just reversed.

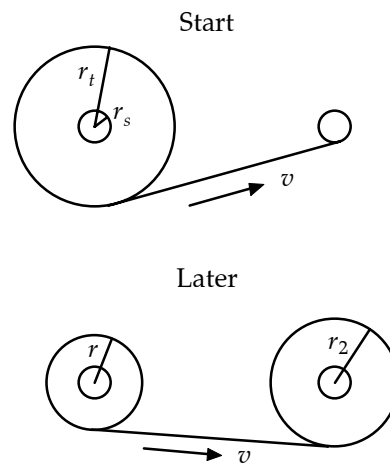


FIG. P10.60

- P10.61** (a) Since only conservative forces act within the system of the rod and the Earth,

$$\Delta E = 0 \quad \text{so} \quad K_f + U_f = K_i + U_i$$

$$\frac{1}{2} I \omega^2 + 0 = 0 + Mg \left(\frac{L}{2} \right)$$

where $I = \frac{1}{3} ML^2$

Therefore,

$$\omega = \sqrt{\frac{3g}{L}}$$

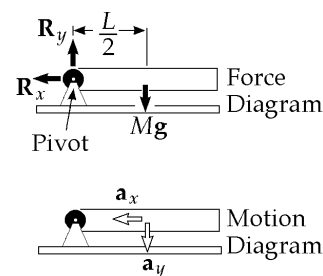


FIG. P10.61

- (b) $\sum \tau = I\alpha$, so that in the horizontal orientation,

$$Mg \left(\frac{L}{2} \right) = \frac{ML^2}{3} \alpha$$

$$\alpha = \sqrt{\frac{3g}{2L}}$$

- (c) $a_x = a_r = -r\omega^2 = -\left(\frac{L}{2}\right)\omega^2 = \boxed{-\frac{3g}{2}}$ $a_y = -a_t = -r\alpha = -\alpha\left(\frac{L}{2}\right) = \boxed{-\frac{3g}{4}}$

- (d) Using Newton's second law, we have

$$R_x = Ma_x = \boxed{-\frac{3Mg}{2}}$$

$$R_y - Mg = Ma_y = -\frac{3Mg}{4} \quad R_y = \boxed{\frac{Mg}{4}}$$

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P10.62 $\alpha = -10.0 \text{ rad/s}^2 - (5.00 \text{ rad/s}^3)t = \frac{d\omega}{dt}$

$$\int_{65.0}^{\omega} d\omega = \int_0^t [-10.0 - 5.00t] dt = -10.0t - 2.50t^2 = \omega - 65.0 \text{ rad/s}$$

$$\omega = \frac{d\theta}{dt} = 65.0 \text{ rad/s} - (10.0 \text{ rad/s}^2)t - (2.50 \text{ rad/s}^3)t^2$$

(a) At $t = 3.00 \text{ s}$,

$$\omega = 65.0 \text{ rad/s} - (10.0 \text{ rad/s}^2)(3.00 \text{ s}) - (2.50 \text{ rad/s}^3)(9.00 \text{ s}^2) = \boxed{12.5 \text{ rad/s}}$$

(b) $\int_0^{\theta} d\theta = \int_0^t \omega dt = \int_0^t [65.0 \text{ rad/s} - (10.0 \text{ rad/s}^2)t - (2.50 \text{ rad/s}^3)t^2] dt$

$$\theta = (65.0 \text{ rad/s})t - (5.00 \text{ rad/s}^2)t^2 - (0.833 \text{ rad/s}^3)t^3$$

At $t = 3.00 \text{ s}$,

$$\theta = (65.0 \text{ rad/s})(3.00 \text{ s}) - (5.00 \text{ rad/s}^2)9.00 \text{ s}^2 - (0.833 \text{ rad/s}^3)27.0 \text{ s}^3$$

$$\theta = \boxed{128 \text{ rad}}$$

P10.63 The first drop has a velocity leaving the wheel given by $\frac{1}{2}mv_i^2 = mgh_1$, so

$$v_1 = \sqrt{2gh_1} = \sqrt{2(9.80 \text{ m/s}^2)(0.540 \text{ m})} = 3.25 \text{ m/s}$$

The second drop has a velocity given by

$$v_2 = \sqrt{2gh_2} = \sqrt{2(9.80 \text{ m/s}^2)(0.510 \text{ m})} = 3.16 \text{ m/s}$$

From $\omega = \frac{v}{r}$, we find

$$\omega_1 = \frac{v_1}{r} = \frac{3.25 \text{ m/s}}{0.381 \text{ m}} = 8.53 \text{ rad/s} \text{ and } \omega_2 = \frac{v_2}{r} = \frac{3.16 \text{ m/s}}{0.381 \text{ m}} = 8.29 \text{ rad/s}$$

or

$$\alpha = \frac{\omega_2^2 - \omega_1^2}{2\theta} = \frac{(8.29 \text{ rad/s})^2 - (8.53 \text{ rad/s})^2}{4\pi} = \boxed{-0.322 \text{ rad/s}^2}$$

- P10.64** At the instant it comes off the wheel, the first drop has a velocity v_1 , directed upward. The magnitude of this velocity is found from

$$K_i + U_{gi} = K_f + U_{gf}$$

$$\frac{1}{2}mv_1^2 + 0 = 0 + mgh_1 \text{ or } v_1 = \sqrt{2gh_1}$$

and the angular velocity of the wheel at the instant the first drop leaves is

$$\omega_1 = \frac{v_1}{R} = \sqrt{\frac{2gh_1}{R^2}}.$$

Similarly for the second drop: $v_2 = \sqrt{2gh_2}$ and $\omega_2 = \frac{v_2}{R} = \sqrt{\frac{2gh_2}{R^2}}.$

The angular acceleration of the wheel is then

$$a = \frac{\omega_2^2 - \omega_1^2}{2\theta} = \frac{\frac{2gh_2}{R^2} - \frac{2gh_1}{R^2}}{2(2\pi)} = \boxed{\frac{g(h_2 - h_1)}{2\pi R^2}}.$$

- P10.65** $K_f = \frac{1}{2}Mv_f^2 + \frac{1}{2}I\omega_f^2$; $U_f = Mgh_f = 0$; $K_i = \frac{1}{2}Mv_i^2 + \frac{1}{2}I\omega_i^2 = 0$
 $U_i = (Mgh)_i$; $f = \mu N = \mu Mg \cos \theta$; $\omega = \frac{v}{r}$; $h = d \sin \theta$ and $I = \frac{1}{2}mr^2$

(a) $\Delta E = E_f - E_i$ or $-fd = K_f + U_f - K_i - U_i$

$$-fd = \frac{1}{2}Mv_f^2 + \frac{1}{2}I\omega_f^2 - Mgh$$

$$-(\mu Mg \cos \theta)d = \frac{1}{2}Mv^2 + \left(\frac{mr^2}{2}\right)\frac{v^2}{r^2} - Mgd \sin \theta$$

$$\frac{1}{2}\left[M + \frac{m}{2}\right]v^2 = Mgd \sin \theta - (\mu Mg \cos \theta)d \text{ or}$$

$$v^2 = 2Mgd \frac{(\sin \theta - \mu \cos \theta)}{\frac{m}{2} + M}$$

$$v_d = \left[4gd \frac{M}{(m + 2M)}(\sin \theta - \mu \cos \theta)\right]^{1/2}$$

(b) $v_f^2 = v_i^2 + 2a\Delta x$, $v_d^2 = 2ad$

$$a = \frac{v_d^2}{2d} = \boxed{2g\left(\frac{M}{m + 2M}\right)(\sin \theta - \mu \cos \theta)}$$

$$\text{P10.66} \quad (a) \quad E = \frac{1}{2} \left(\frac{2}{5} MR^2 \right) (\omega^2)$$

$$E = \frac{1}{2} \cdot \frac{2}{5} (5.98 \times 10^{24}) (6.37 \times 10^6)^2 \left(\frac{2\pi}{86400} \right)^2 = \boxed{2.57 \times 10^{29} \text{ J}}$$

$$\begin{aligned} (b) \quad \frac{dE}{dt} &= \frac{d}{dt} \left[\frac{1}{2} \left(\frac{2}{5} MR^2 \right) \left(\frac{2\pi}{T} \right)^2 \right] \\ &= \frac{1}{5} MR^2 (2\pi)^2 (-2T^{-3}) \frac{dT}{dt} \\ &= \frac{1}{5} MR^2 \left(\frac{2\pi}{T} \right)^2 \left(\frac{-2}{T} \right) \frac{dT}{dt} \\ &= (2.57 \times 10^{29} \text{ J}) \left(\frac{-2}{86400 \text{ s}} \right) \left(\frac{10 \times 10^{-6} \text{ s}}{3.16 \times 10^7 \text{ s}} \right) (86400 \text{ s/day}) \\ \frac{dE}{dt} &= \boxed{-1.63 \times 10^{17} \text{ J/day}} \end{aligned}$$

$$\text{*P10.67} \quad (a) \quad \omega_f = \omega_i + \alpha t$$

$$\begin{aligned} \alpha &= \frac{\omega_f - \omega_i}{t} = \frac{\frac{2\pi}{T_f} - \frac{2\pi}{T_i}}{t} = \frac{2\pi(T_i - T_f)}{T_i T_f t} \\ &\sim \frac{2\pi(-10^{-3} \text{ s})}{1 \text{ d } 1 \text{ d } 100 \text{ yr}} \left(\frac{1 \text{ d}}{86400 \text{ s}} \right)^2 \left(\frac{1 \text{ yr}}{3.156 \times 10^7 \text{ s}} \right) = \boxed{-10^{-22} \text{ s}^{-2}} \end{aligned}$$

(b) The Earth, assumed uniform, has moment of inertia

$$\begin{aligned} I &= \frac{2}{5} MR^2 = \frac{2}{5} (5.98 \times 10^{24} \text{ kg}) (6.37 \times 10^6 \text{ m})^2 = 9.71 \times 10^{37} \text{ kg} \cdot \text{m}^2 \\ \sum \tau &= I\alpha \sim 9.71 \times 10^{37} \text{ kg} \cdot \text{m}^2 (-2.67 \times 10^{-22} \text{ s}^{-2}) = \boxed{-10^{16} \text{ N} \cdot \text{m}} \end{aligned}$$

The negative sign indicates clockwise, to slow the planet's counterclockwise rotation.

(c) $|\tau| = Fd$. Suppose the person can exert a 900-N force.

$$d = \frac{|\tau|}{F} = \frac{2.59 \times 10^{16} \text{ N} \cdot \text{m}}{900 \text{ N}} \sim \boxed{10^{13} \text{ m}}$$

This is the order of magnitude of the size of the planetary system.

P10.68 $\Delta\theta = \omega t$

$$t = \frac{\Delta\theta}{\omega} = \frac{\left(\frac{31.0^\circ}{360^\circ}\right) \text{ rev}}{\frac{900 \text{ rev}}{60 \text{ s}}} = 0.00574 \text{ s}$$

$$v = \frac{0.800 \text{ m}}{0.00574 \text{ s}} = \boxed{139 \text{ m/s}}$$

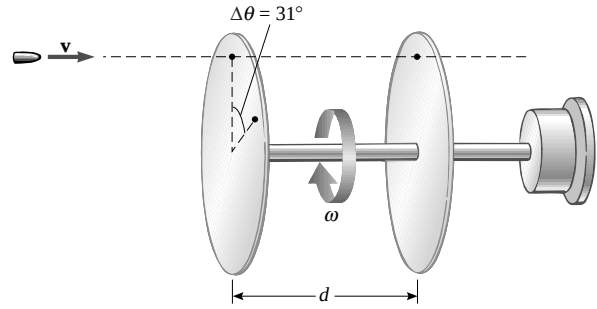


FIG. P10.68

P10.69 τ_f will oppose the torque due to the hanging object:

$$\sum \tau = I\alpha = TR - \tau_f: \quad \tau_f = TR - I\alpha \quad (1)$$

Now find T , I and α in given or known terms and substitute into equation (1).

$$\sum F_y = T - mg = -ma: \quad T = m(g - a) \quad (2)$$

$$\text{also } \Delta y = v_i t + \frac{at^2}{2} \quad a = \frac{2y}{t^2} \quad (3)$$

$$\text{and} \quad \alpha = \frac{a}{R} = \frac{2y}{Rt^2}: \quad (4)$$

$$I = \frac{1}{2}M \left[R^2 + \left(\frac{R}{2} \right)^2 \right] = \frac{5}{8}MR^2 \quad (5)$$

Substituting (2), (3), (4), and (5) into (1),

$$\text{we find} \quad \tau_f = m \left(g - \frac{2y}{t^2} \right) R - \frac{5}{8} \frac{MR^2(2y)}{Rt^2} = \boxed{R \left[m \left(g - \frac{2y}{t^2} \right) - \frac{5}{4} \frac{My}{t^2} \right]}$$

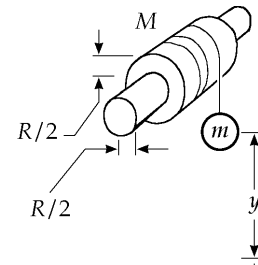


FIG. P10.69

P10.70 (a) $W = \Delta K + \Delta U$

$$W = K_f - K_i + U_f - U_i$$

$$0 = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 - mgd \sin \theta - \frac{1}{2}kd^2$$

$$\frac{1}{2}\omega^2(I + mR^2) = mgd \sin \theta + \frac{1}{2}kd^2$$

$$\boxed{\omega = \sqrt{\frac{2mgd \sin \theta + kd^2}{I + mR^2}}}$$

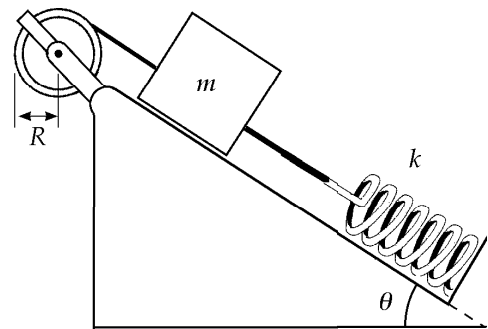


FIG. P10.70

$$(b) \quad \omega = \sqrt{\frac{2(0.500 \text{ kg})(9.80 \text{ m/s}^2)(0.200 \text{ m})(\sin 37.0^\circ) + 50.0 \text{ N/m}(0.200 \text{ m})^2}{1.00 \text{ kg} \cdot \text{m}^2 + 0.500 \text{ kg}(0.300 \text{ m})^2}}$$

$$\omega = \sqrt{\frac{1.18 + 2.00}{1.05}} = \sqrt{3.04} = \boxed{1.74 \text{ rad/s}}$$

P10.71 (a) $m_2 g - T_2 = m_2 a$
 $T_2 = m_2 (g - a) = 20.0 \text{ kg} (9.80 \text{ m/s}^2 - 2.00 \text{ m/s}^2) = \boxed{156 \text{ N}}$
 $T_1 - m_1 g \sin 37.0^\circ = m_1 a$
 $T_1 = (15.0 \text{ kg}) (9.80 \sin 37.0^\circ + 2.00) \text{ m/s}^2 = \boxed{118 \text{ N}}$

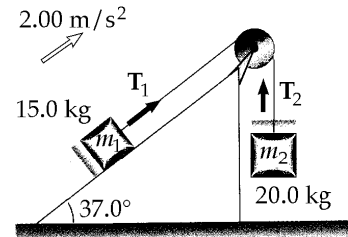


FIG. P10.71

(b) $(T_2 - T_1)R = I\alpha = I\left(\frac{a}{R}\right)$
 $I = \frac{(T_2 - T_1)R^2}{a} = \frac{(156 \text{ N} - 118 \text{ N})(0.250 \text{ m})^2}{2.00 \text{ m/s}^2} = \boxed{1.17 \text{ kg} \cdot \text{m}^2}$

P10.72 For the board just starting to move,

$$\sum \tau = I\alpha: \quad mg\left(\frac{\ell}{2}\right)\cos\theta = \left(\frac{1}{3}m\ell^2\right)\alpha$$

$$\alpha = \frac{3}{2}\left(\frac{g}{\ell}\right)\cos\theta$$

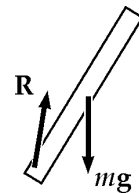


FIG. P10.72

The tangential acceleration of the end is $a_t = \ell\alpha = \frac{3}{2}g\cos\theta$

The vertical component is $a_y = a_t \cos\theta = \frac{3}{2}g\cos^2\theta$

If this is greater than g , the board will pull ahead of the ball falling:

(a) $\frac{3}{2}g\cos^2\theta \geq g$ gives $\cos^2\theta \geq \frac{2}{3}$ so $\cos\theta \geq \sqrt{\frac{2}{3}}$ and $\boxed{\theta \leq 35.3^\circ}$

(b) When $\theta = 35.3^\circ$, the cup will land underneath the release-point of the ball if $r_c = \ell \cos\theta$

When $\ell = 1.00 \text{ m}$, and $\theta = 35.3^\circ$ $r_c = 1.00 \text{ m} \sqrt{\frac{2}{3}} = 0.816 \text{ m}$

so the cup should be $(1.00 \text{ m} - 0.816 \text{ m}) = \boxed{0.184 \text{ m from the moving end}}$

P10.73 At $t = 0$, $\omega = 3.50 \text{ rad/s} = \omega_0 e^0$. Thus, $\omega_0 = 3.50 \text{ rad/s}$

At $t = 9.30 \text{ s}$, $\omega = 2.00 \text{ rad/s} = \omega_0 e^{-\sigma(9.30 \text{ s})}$, yielding $\sigma = 6.02 \times 10^{-2} \text{ s}^{-1}$

(a) $\alpha = \frac{d\omega}{dt} = \frac{d(\omega_0 e^{-\sigma t})}{dt} = \omega_0 (-\sigma) e^{-\sigma t}$
 At $t = 3.00 \text{ s}$,
 $\alpha = (3.50 \text{ rad/s})(-6.02 \times 10^{-2} \text{ s}^{-1})e^{-3.00(6.02 \times 10^{-2})} = \boxed{-0.176 \text{ rad/s}^2}$

(b) $\theta = \int_0^t \omega_0 e^{-\sigma t} dt = \frac{\omega_0}{-\sigma} [e^{-\sigma t} - 1] = \frac{\omega_0}{\sigma} [1 - e^{-\sigma t}]$
 At $t = 2.50 \text{ s}$,
 $\theta = \frac{3.50 \text{ rad/s}}{(6.02 \times 10^{-2}) 1/\text{s}} [1 - e^{-(6.02 \times 10^{-2})(2.50)}] = 8.12 \text{ rad} = \boxed{1.29 \text{ rev}}$

(c) As $t \rightarrow \infty$, $\theta \rightarrow \frac{\omega_0}{\sigma} (1 - e^{-\infty}) = \frac{3.50 \text{ rad/s}}{6.02 \times 10^{-2} \text{ s}^{-1}} = 58.2 \text{ rad} = \boxed{9.26 \text{ rev}}$

P10.74 Consider the total weight of each hand to act at the center of gravity (mid-point) of that hand. Then the total torque (taking CCW as positive) of these hands about the center of the clock is given by

$$\tau = -m_h g \left(\frac{L_h}{2} \right) \sin \theta_h - m_m g \left(\frac{L_m}{2} \right) \sin \theta_m = -\frac{g}{2} (m_h L_h \sin \theta_h + m_m L_m \sin \theta_m)$$

If we take $t = 0$ at 12 o'clock, then the angular positions of the hands at time t are

$$\theta_h = \omega_h t,$$

where

$$\omega_h = \frac{\pi}{6} \text{ rad/h}$$

and

$$\theta_m = \omega_m t,$$

where

$$\omega_m = 2\pi \text{ rad/h}$$

Therefore,

$$\tau = -4.90 \text{ m/s}^2 \left[60.0 \text{ kg}(2.70 \text{ m}) \sin \left(\frac{\pi t}{6} \right) + 100 \text{ kg}(4.50 \text{ m}) \sin 2\pi t \right]$$

or

$$\tau = -794 \text{ N} \cdot \text{m} \left[\sin \left(\frac{\pi t}{6} \right) + 2.78 \sin 2\pi t \right], \text{ where } t \text{ is in hours.}$$

(a) (i) At 3:00, $t = 3.00 \text{ h}$,

$$\text{so } \tau = -794 \text{ N} \cdot \text{m} \left[\sin \left(\frac{\pi}{2} \right) + 2.78 \sin 6\pi \right] = \boxed{-794 \text{ N} \cdot \text{m}}$$

(ii) At 5:15, $t = 5 \text{ h} + \frac{15}{60} \text{ h} = 5.25 \text{ h}$, and substitution gives:

$$\tau = \boxed{-2510 \text{ N} \cdot \text{m}}$$

(iii) At 6:00, $\tau = \boxed{0 \text{ N} \cdot \text{m}}$

(iv) At 8:20, $\tau = \boxed{-1160 \text{ N} \cdot \text{m}}$

(v) At 9:45, $\tau = \boxed{-2940 \text{ N} \cdot \text{m}}$

(b) The total torque is zero at those times when

$$\sin \left(\frac{\pi t}{6} \right) + 2.78 \sin 2\pi t = 0$$

We proceed numerically, to find 0, 0.515 295 5, ..., corresponding to the times

12:00:00	12:30:55	12:58:19	1:32:31	1:57:01
2:33:25	2:56:29	3:33:22	3:56:55	4:32:24
4:58:14	5:30:52	6:00:00	6:29:08	7:01:46
7:27:36	8:03:05	8:26:38	9:03:31	9:26:35
10:02:59	10:27:29	11:01:41	11:29:05	

- *P10.75** (a) As the bicycle frame moves forward at speed v , the center of each wheel moves forward at the same speed and the wheels turn at angular speed $\omega = \frac{v}{R}$. The total kinetic energy of the bicycle is

$$K = K_{\text{trans}} + K_{\text{rot}}$$

or

$$K = \frac{1}{2}(m_{\text{frame}} + 2m_{\text{wheel}})v^2 + 2\left(\frac{1}{2}I_{\text{wheel}}\omega^2\right) = \frac{1}{2}(m_{\text{frame}} + 2m_{\text{wheel}})v^2 + \left(\frac{1}{2}m_{\text{wheel}}R^2\right)\left(\frac{v^2}{R^2}\right).$$

This yields

$$K = \frac{1}{2}(m_{\text{frame}} + 3m_{\text{wheel}})v^2 = \frac{1}{2}[8.44 \text{ kg} + 3(0.820 \text{ kg})](3.35 \text{ m/s})^2 = \boxed{61.2 \text{ J}}.$$

- (b) As the block moves forward with speed v , the top of each trunk moves forward at the same speed and the center of each trunk moves forward at speed $\frac{v}{2}$. The angular speed of each roller is $\omega = \frac{v}{2R}$. As in part (a), we have one object undergoing pure translation and two identical objects rolling without slipping. The total kinetic energy of the system of the stone and the trees is

$$K = K_{\text{trans}} + K_{\text{rot}}$$

or

$$K = \frac{1}{2}m_{\text{stone}}v^2 + 2\left(\frac{1}{2}m_{\text{tree}}\left(\frac{v}{2}\right)^2\right) + 2\left(\frac{1}{2}I_{\text{tree}}\omega^2\right) = \frac{1}{2}\left(m_{\text{stone}} + \frac{1}{2}m_{\text{tree}}\right)v^2 + \left(\frac{1}{2}m_{\text{tree}}R^2\right)\left(\frac{v^2}{4R^2}\right).$$

This gives

$$K = \frac{1}{2}\left(m_{\text{stone}} + \frac{3}{4}m_{\text{tree}}\right)v^2 = \frac{1}{2}[844 \text{ kg} + 0.75(82.0 \text{ kg})](0.335 \text{ m/s})^2 = \boxed{50.8 \text{ J}}.$$

- P10.76** Energy is conserved so $\Delta U + \Delta K_{\text{rot}} + \Delta K_{\text{trans}} = 0$

$$mg(R-r)(\cos\theta - 1) + \left[\frac{1}{2}mv^2 - 0\right] + \frac{1}{2}\left[\frac{2}{5}mr^2\right]\omega^2 = 0$$

Since $r\omega = v$, this gives

$$\omega = \sqrt{\frac{10(R-r)(1-\cos\theta)g}{7r^2}}$$

or $\omega = \boxed{\sqrt{\frac{10Rg(1-\cos\theta)}{7r^2}}}$ since $R \gg r$.

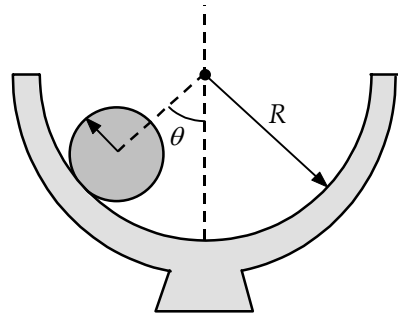


FIG. P10.76

P10.77 $\sum F = T - Mg = -Ma$; $\sum \tau = TR = I\alpha = \frac{1}{2}MR^2\left(\frac{a}{R}\right)$

(a) Combining the above two equations we find

$$T = M(g - a)$$

and

$$a = \frac{2T}{M}$$

thus

$$T = \boxed{\frac{Mg}{3}}$$

(b) $a = \frac{2T}{M} = \frac{2}{M}\left(\frac{Mg}{3}\right) = \boxed{\frac{2}{3}g}$

(c) $v_f^2 = v_i^2 + 2a(x_f - x_i)$ $v_f^2 = 0 + 2\left(\frac{2}{3}g\right)(h - 0)$

$$v_f = \boxed{\sqrt{\frac{4gh}{3}}}$$

For comparison, from conservation of energy for the system of the disk and the Earth we have

$$U_{gi} + K_{rot i} + K_{trans i} = U_{gf} + K_{rot f} + K_{trans f}: \quad Mgh + 0 + 0 = 0 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v_f}{R}\right)^2 + \frac{1}{2}Mv_f^2$$

$$v_f = \sqrt{\frac{4gh}{3}}$$

P10.78 (a) $\sum F_x = F - f = Ma$; $\sum \tau = fR = I\alpha$

Using $I = \frac{1}{2}MR^2$ and $\alpha = \frac{a}{R}$, we find $a = \boxed{\frac{2F}{3M}}$

(b) When there is no slipping, $f = \mu Mg$.

Substituting this into the torque equation of part (a), we have

$$\mu MgR = \frac{1}{2}MRa \text{ and } \mu = \boxed{\frac{F}{3Mg}}.$$

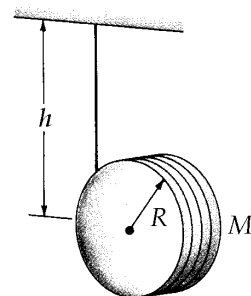


FIG. P10.77

316 Rotation of a Rigid Object About a Fixed Axis

P10.79 (a) $\Delta K_{\text{rot}} + \Delta K_{\text{trans}} + \Delta U = 0$

Note that initially the center of mass of the sphere is a distance $h + r$ above the bottom of the loop; and as the mass reaches the top of the loop, this distance above the reference level is $2R - r$. The conservation of energy requirement gives

$$mg(h + r) = mg(2R - r) + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

For the sphere $I = \frac{2}{5}mr^2$ and $v = r\omega$ so that the expression becomes

$$gh + 2gr = 2gR + \frac{7}{10}v^2 \quad (1)$$

Note that $h = h_{\text{min}}$ when the speed of the sphere at the top of the loop satisfies the condition

$$\sum F = mg = \frac{mv^2}{(R - r)} \text{ or } v^2 = g(R - r)$$

Substituting this into Equation (1) gives

$$h_{\text{min}} = 2(R - r) + 0.700(R - r) \text{ or } \boxed{h_{\text{min}} = 2.70(R - r) = 2.70R}$$

- (b) When the sphere is initially at $h = 3R$ and finally at point P , the conservation of energy equation gives

$$mg(3R + r) = mgR + \frac{1}{2}mv^2 + \frac{1}{5}mv^2, \text{ or}$$

$$v^2 = \frac{10}{7}(2R + r)g$$

Turning clockwise as it rolls without slipping past point P , the sphere is slowing down with counterclockwise angular acceleration caused by the torque of an upward force f of static friction. We have $\sum F_y = ma_y$ and $\sum \tau = I\alpha$ becoming $f - mg = -m\alpha r$ and $fr = \left(\frac{2}{5}\right)mr^2\alpha$.

Eliminating f by substitution yields $\alpha = \frac{5g}{7r}$ so that $\sum F_y = \boxed{-\frac{5}{7}mg}$

$$\sum F_x = -n = -\frac{mv^2}{R - r} = -\frac{\left(\frac{10}{7}\right)(2R + r)}{R - r}mg = \boxed{\frac{-20mg}{7}} \text{ (since } R \gg r \text{)}$$

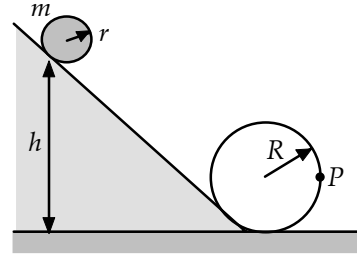


FIG. P10.79

P10.80 Consider the free-body diagram shown. The sum of torques about the chosen pivot is

$$\sum \tau = I\alpha \Rightarrow F\ell = \left(\frac{1}{3}ml^2\right)\left(\frac{a_{\text{CM}}}{\frac{l}{2}}\right) = \left(\frac{2}{3}ml\right)a_{\text{CM}} \quad (1)$$

(a) $\ell = l = 1.24 \text{ m}$: In this case, Equation (1) becomes

$$a_{\text{CM}} = \frac{3F}{2m} = \frac{3(14.7 \text{ N})}{2(0.630 \text{ kg})} = 35.0 \text{ m/s}^2$$

$$\sum F_x = ma_{\text{CM}} \Rightarrow F + H_x = ma_{\text{CM}} \text{ or } H_x = ma_{\text{CM}} - F$$

$$\text{Thus, } H_x = (0.630 \text{ kg})(35.0 \text{ m/s}^2) - 14.7 \text{ N} = +7.35 \text{ N or}$$

$$\mathbf{H}_x = \boxed{7.35\hat{\mathbf{i}} \text{ N}}.$$

(b) $\ell = \frac{1}{2}l = 0.620 \text{ m}$: For this situation, Equation (1) yields

$$a_{\text{CM}} = \frac{3F}{4m} = \frac{3(14.7 \text{ N})}{4(0.630 \text{ kg})} = 17.5 \text{ m/s}^2.$$

$$\text{Again, } \sum F_x = ma_{\text{CM}} \Rightarrow H_x = ma_{\text{CM}} - F, \text{ so}$$

$$H_x = (0.630 \text{ kg})(17.5 \text{ m/s}^2) - 14.7 \text{ N} = -3.68 \text{ N or } \mathbf{H}_x = \boxed{-3.68\hat{\mathbf{i}} \text{ N}}.$$

(c) If $H_x = 0$, then $\sum F_x = ma_{\text{CM}} \Rightarrow F = ma_{\text{CM}}$, or $a_{\text{CM}} = \frac{F}{m}$.

Thus, Equation (1) becomes

$$F\ell = \left(\frac{2}{3}ml\right)\left(\frac{F}{m}\right) \text{ so } \ell = \frac{2}{3}l = \frac{2}{3}(1.24 \text{ m}) = \boxed{0.827 \text{ m (from the top)}}.$$

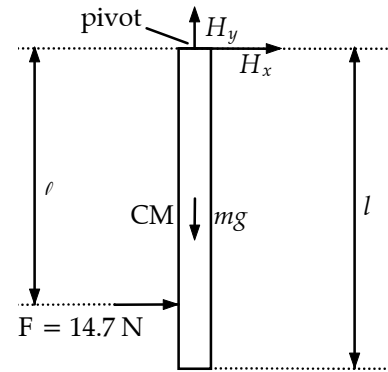


FIG. P10.80

P10.81 Let the ball have mass m and radius r . Then $I = \frac{2}{5}mr^2$. If the ball takes four seconds to go down twenty-meter alley, then $\bar{v} = 5 \text{ m/s}$. The translational speed of the ball will decrease somewhat as the ball loses energy to sliding friction and some translational kinetic energy is converted to rotational kinetic energy; but its speed will always be on the order of 5.00 m/s , including at the starting point.

As the ball slides, the kinetic friction force exerts a torque on the ball to increase the angular speed. When $\omega = \frac{v}{r}$, the ball has achieved pure rolling motion, and kinetic friction ceases. To determine the elapsed time before pure rolling motion is achieved, consider:

$$\sum \tau = I\alpha \Rightarrow (\mu_k mg)r = \left(\frac{2}{5}mr^2\right)\left[\frac{(5.00 \text{ m/s})/r}{t}\right] \text{ which gives}$$

$$t = \frac{2(5.00 \text{ m/s})}{5\mu_k g} = \frac{2.00 \text{ m/s}}{\mu_k g}$$

Note that the mass and radius of the ball have canceled. If $\mu_k = 0.100$ for the polished alley, the sliding distance will be given by

$$\Delta x = \bar{v}t = (5.00 \text{ m/s})\left[\frac{2.00 \text{ m/s}}{(0.100)(9.80 \text{ m/s}^2)}\right] = 10.2 \text{ m or } \Delta x \sim \boxed{10^1 \text{ m}}.$$

P10.82 Conservation of energy between apex and the point where the grape leaves the surface:

$$mg\Delta y = \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2$$

$$mgR(1 - \cos\theta) = \frac{1}{2}mv_f^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(\frac{v_f}{R}\right)^2$$

which gives $g(1 - \cos\theta) = \frac{7}{10}\left(\frac{v_f^2}{R}\right)$ (1)

Consider the radial forces acting on the grape:

$$mg \cos\theta - n = \frac{mv_f^2}{R}.$$

At the point where the grape leaves the surface, $n \rightarrow 0$.

Thus, $mg \cos\theta = \frac{mv_f^2}{R}$ or $\frac{v_f^2}{R} = g \cos\theta$.

Substituting this into Equation (1) gives

$$g - g \cos\theta = \frac{7}{10}g \cos\theta \text{ or } \cos\theta = \frac{10}{17} \text{ and } \theta = \boxed{54.0^\circ}.$$

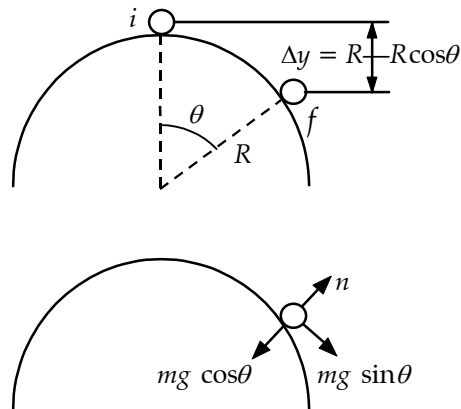


FIG. P10.82

P10.83 (a) There are not any horizontal forces acting on the rod, so the center of mass will not move horizontally. Rather, the center of mass drops straight downward (distance $h/2$) with the rod rotating about the center of mass as it falls. From conservation of energy:

$$K_f + U_{gf} = K_i + U_{gi}$$

$$\frac{1}{2}Mv_{\text{CM}}^2 + \frac{1}{2}I\omega^2 + 0 = 0 + Mg\left(\frac{h}{2}\right) \text{ or}$$

$$\frac{1}{2}Mv_{\text{CM}}^2 + \frac{1}{2}\left(\frac{1}{12}Mh^2\right)\left(\frac{v_{\text{CM}}}{\frac{h}{2}}\right)^2 = Mg\left(\frac{h}{2}\right) \text{ which reduces to}$$

$$v_{\text{CM}} = \boxed{\sqrt{\frac{3gh}{4}}}$$

(b) In this case, the motion is a pure rotation about a fixed pivot point (the lower end of the rod) with the center of mass moving in a circular path of radius $h/2$. From conservation of energy:

$$K_f + U_{gf} = K_i + U_{gi}$$

$$\frac{1}{2}I\omega^2 + 0 = 0 + Mg\left(\frac{h}{2}\right) \text{ or}$$

$$\frac{1}{2}\left(\frac{1}{3}Mh^2\right)\left(\frac{v_{\text{CM}}}{\frac{h}{2}}\right)^2 = Mg\left(\frac{h}{2}\right) \text{ which reduces to}$$

$$v_{\text{CM}} = \boxed{\sqrt{\frac{3gh}{4}}}$$

- P10.84** (a) The mass of the roll decreases as it unrolls. We have $m = \frac{Mr^2}{R^2}$ where M is the initial mass of the roll. Since $\Delta E = 0$, we then have $\Delta U_g + \Delta K_{\text{trans}} + \Delta K_{\text{rot}} = 0$. Thus, when $I = \frac{mr^2}{2}$,

$$(mgr - MgR) + \frac{mv^2}{2} + \left[\frac{mr^2}{2} \frac{\omega^2}{2} \right] = 0$$

Since $\omega r = v$, this becomes $v = \sqrt{\frac{4g(R^3 - r^3)}{3r^2}}$

- (b) Using the given data, we find $v = \boxed{5.31 \times 10^4 \text{ m/s}}$
- (c) We have assumed that $\Delta E = 0$. When the roll gets to the end, we will have an inelastic collision with the surface. The energy goes into internal energy. With the assumption we made, there are problems with this question. It would take an infinite time to unwrap the tissue since $dr \rightarrow 0$. Also, as r approaches zero, the velocity of the center of mass approaches infinity, which is physically impossible.

- P10.85** (a) $\sum F_x = F + f = Ma_{\text{CM}}$
 $\sum \tau = FR - fR = I\alpha$
 $FR - (Ma_{\text{CM}} - F)R = \frac{Ia_{\text{CM}}}{R}$

$$\boxed{a_{\text{CM}} = \frac{4F}{3M}}$$

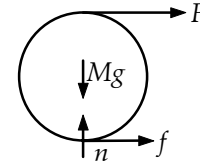


FIG. P10.85

- (b) $f = Ma_{\text{CM}} - F = M\left(\frac{4F}{3M}\right) - F = \boxed{\frac{1}{3}F}$
- (c) $v_f^2 = v_i^2 + 2a(x_f - x_i)$
 $v_f = \boxed{\sqrt{\frac{8Fd}{3M}}}$

P10.86 Call f_t the frictional force exerted by each roller backward on the plank. Name as f_b the rolling resistance exerted backward by the ground on each roller. Suppose the rollers are equally far from the ends of the plank.

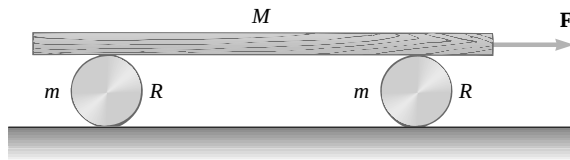


FIG. P10.86

For the plank,

$$\sum F_x = ma_x \quad 6.00 \text{ N} - 2f_t = (6.00 \text{ kg})a_p$$

The center of each roller moves forward only half as far as the plank. Each roller has acceleration $\frac{a_p}{2}$ and angular acceleration

$$\frac{a_p/2}{(5.00 \text{ cm})} = \frac{a_p}{(0.100 \text{ m})}$$

Then for each,

$$\sum F_x = ma_x \quad +f_t - f_b = (2.00 \text{ kg})\frac{a_p}{2}$$

$$\sum \tau = I\alpha \quad f_t(5.00 \text{ cm}) + f_b(5.00 \text{ cm}) = \frac{1}{2}(2.00 \text{ kg})(5.00 \text{ cm})^2 \frac{a_p}{10.0 \text{ cm}}$$

So $f_t + f_b = \left(\frac{1}{2} \text{ kg}\right)a_p$

Add to eliminate f_b :

$$2f_t = (1.50 \text{ kg})a_p$$

(a) And $6.00 \text{ N} - (1.50 \text{ kg})a_p = (6.00 \text{ kg})a_p$

$$a_p = \frac{(6.00 \text{ N})}{(7.50 \text{ kg})} = \boxed{0.800 \text{ m/s}^2}$$

For each roller, $a = \frac{a_p}{2} = \boxed{0.400 \text{ m/s}^2}$

(b) Substituting back, $2f_t = (1.50 \text{ kg})0.800 \text{ m/s}^2$

$$f_t = \boxed{0.600 \text{ N}}$$

$$0.600 \text{ N} + f_b = \frac{1}{2} \text{ kg}(0.800 \text{ m/s}^2)$$

$$f_b = -0.200 \text{ N}$$

The negative sign means that the horizontal force of ground on each roller is $\boxed{0.200 \text{ N forward}}$ rather than backward as we assumed.

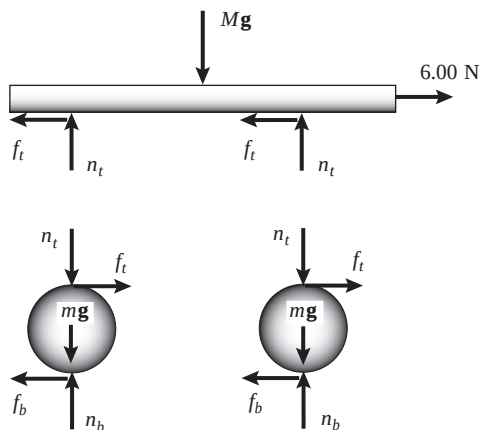


FIG. P10.86(b)

- P10.87** Rolling is instantaneous rotation about the contact point P . The weight and normal force produce no torque about this point.

Now F_1 produces a clockwise torque about P and makes the spool roll forward.

Counterclockwise torques result from F_3 and F_4 , making the spool roll to the left.

The force F_2 produces zero torque about point P and does not cause the spool to roll. If F_2 were strong enough, it would cause the spool to slide to the right, but not roll.

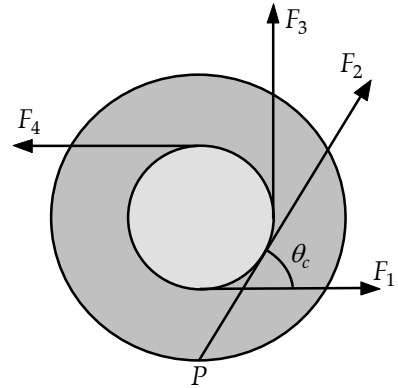


FIG. P10.87

- P10.88** The force applied at the critical angle exerts zero torque about the spool's contact point with the ground and so will not make the spool roll.

From the right triangle shown in the sketch, observe that $\theta_c = 90^\circ - \phi = 90^\circ - (90^\circ - \gamma) = \gamma$.

Thus, $\cos \theta_c = \cos \gamma = \frac{r}{R}$.

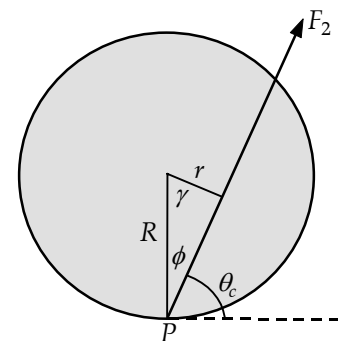


FIG. P10.88

- P10.89** (a) Consider motion starting from rest over distance x along the incline:

$$\begin{aligned} (K_{\text{trans}} + K_{\text{rot}} + U)_i + \Delta E &= (K_{\text{trans}} + K_{\text{rot}} + U)_f \\ 0 + 0 + Mgx \sin \theta + 0 &= \frac{1}{2} Mv^2 + 2 \left(\frac{1}{2} mR^2 \right) \left(\frac{v}{R} \right)^2 + 0 \\ 2Mgx \sin \theta &= (M + 2m)v^2 \end{aligned}$$

Since acceleration is constant,

$$v^2 = v_i^2 + 2ax = 0 + 2ax, \text{ so}$$

$$2Mgx \sin \theta = (M + 2m)2ax$$

$$a = \frac{Mg \sin \theta}{(M + 2m)}$$

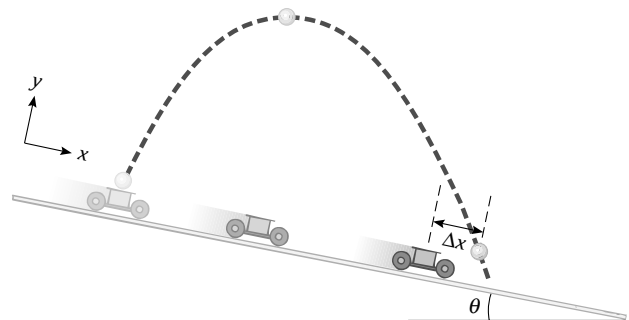


FIG. P10.88

continued on next page

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- (c) Suppose the ball is fired from a cart at rest. It moves with acceleration $g \sin \theta = a_x$ down the incline and $a_y = -g \cos \theta$ perpendicular to the incline. For its range along the ramp, we have

$$y - y_i = v_{yi}t - \frac{1}{2}g \cos \theta t^2 = 0 - 0$$

$$t = \frac{2v_{yi}}{g \cos \theta}$$

$$x - x_i = v_{xi}t + \frac{1}{2}a_x t^2$$

$$d = 0 + \frac{1}{2}g \sin \theta \left(\frac{4v_{yi}^2}{g^2 \cos^2 \theta} \right)$$

$$d = \boxed{\frac{2v_{yi}^2 \sin \theta}{g \cos^2 \theta}}$$

- (b) In the same time the cart moves

$$x - x_i = v_{xi}t + \frac{1}{2}a_x t^2$$

$$d_c = 0 + \frac{1}{2} \left(\frac{g \sin \theta M}{(M + 2m)} \right) \left(\frac{4v_{yi}^2}{g^2 \cos^2 \theta} \right)$$

$$d_c = \frac{2v_{yi}^2 \sin \theta M}{g(M + 2m) \cos^2 \theta}$$

So the ball overshoots the cart by

$$\Delta x = d - d_c = \frac{2v_{yi}^2 \sin \theta}{g \cos^2 \theta} - \frac{2v_{yi}^2 \sin \theta M}{g \cos^2 \theta (M + 2m)}$$

$$\Delta x = \frac{2v_{yi}^2 \sin \theta M + 4v_{yi}^2 \sin \theta m - 2v_{yi}^2 \sin \theta M}{g \cos^2 \theta (M + 2m)}$$

$$\Delta x = \boxed{\frac{4mv_{yi}^2 \sin \theta}{(M + 2m)g \cos^2 \theta}}$$

P10.90 $\sum F_x = ma_x$ reads $-f + T = ma$. If we take torques around the center of mass, we can use $\sum \tau = I\alpha$, which reads $+fR_2 - TR_1 = I\alpha$. For rolling without slipping, $\alpha = \frac{a}{R_2}$. By substitution,

$$fR_2 - TR_1 = \frac{Ia}{R_2} = \frac{I}{R_2 m} (T - f)$$

$$fR_2^2 m - TR_1 R_2 m = IT - If$$

$$f(I + mR_2^2) = T(I + mR_1 R_2)$$

$$f = \left(\frac{I + mR_1 R_2}{I + mR_2^2} \right) T$$

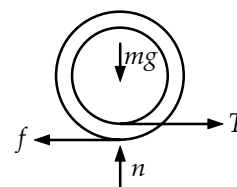


FIG. P10.90

Since the answer is positive, the friction force is confirmed to be to the left.

ANSWERS TO EVEN PROBLEMS

P10.2 (a) 822 rad/s^2 ; (b) $4.21 \times 10^3 \text{ rad}$

P10.4 (a) $1.20 \times 10^2 \text{ rad/s}$; (b) 25.0 s

P10.6 -226 rad/s^2

P10.8 13.7 rad/s^2

P10.10 (a) 2.88 s ; (b) 12.8 s

P10.12 (a) 0.180 rad/s ;
(b) 8.10 m/s^2 toward the center of the track

P10.14 (a) 0.605 m/s ; (b) 17.3 rad/s ; (c) 5.82 m/s ;
(d) The crank length is unnecessary

P10.16 (a) 54.3 rev ; (b) 12.1 rev/s

P10.18 0.572

P10.20 (a) $92.0 \text{ kg} \cdot \text{m}^2$; 184 J ;
(b) 6.00 m/s ; 4.00 m/s ; 8.00 m/s ; 184 J

P10.22 see the solution

P10.24 $1.28 \text{ kg} \cdot \text{m}^2$

P10.26 $\sim 10^0 \text{ kg} \cdot \text{m}^2$

P10.28 $\frac{1}{2} ML^2$

P10.30 $168 \text{ N} \cdot \text{m}$ clockwise

P10.32 $882 \text{ N} \cdot \text{m}$

P10.34 (a) 1.03 s ; (b) 10.3 rev

P10.36 (a) $21.6 \text{ kg} \cdot \text{m}^2$; (b) $3.60 \text{ N} \cdot \text{m}$; (c) 52.4 rev

P10.38 0.312

P10.40 $1.04 \times 10^{-3} \text{ J}$

P10.42 149 rad/s

P10.44 (a) 6.90 J ; (b) 8.73 rad/s ; (c) 2.44 m/s ;
(d) 1.043 2 times larger

P10.46 2.36 m/s

P10.48 276 J

P10.50 (a) 74.3 W ; (b) 401 W

P10.52 $\frac{7Mv^2}{10}$

P10.54 The disk; $\sqrt{\frac{4gh}{3}}$ versus $\sqrt{gh}$

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P10.56 (a) 2.38 m/s; (b) 4.31 m/s;
(c) It will not reach the top of the loop.

P10.58 (a) 0.992 W; (b) 827 W

P10.60 see the solution

P10.62 (a) 12.5 rad/s; (b) 128 rad

P10.64 $\frac{g(h_2 - h_1)}{2\pi R^2}$

P10.66 (a) 2.57×10^{29} J; (b) -1.63×10^{17} J/day

P10.68 139 m/s

P10.70 (a) $\sqrt{\frac{2mgd \sin \theta + kd^2}{I + mR^2}}$; (b) 1.74 rad/s

P10.72 see the solution

P10.74 (a) $-794 \text{ N} \cdot \text{m}$; $-2\,510 \text{ N} \cdot \text{m}$; 0;
 $-1\,160 \text{ N} \cdot \text{m}$; $-2\,940 \text{ N} \cdot \text{m}$;
(b) see the solution

P10.76 $\sqrt{\frac{10Rg(1 - \cos \theta)}{7r^2}}$

P10.78 see the solution

P10.80 (a) 35.0 m/s^2 ; $7.35\hat{\mathbf{i}} \text{ N}$;
(b) 17.5 m/s^2 ; $-3.68\hat{\mathbf{i}} \text{ N}$;
(c) At 0.827 m from the top.

P10.82 54.0°

P10.84 (a) $\sqrt{\frac{4g(R^3 - r^3)}{3r^2}}$; (b) $5.31 \times 10^4 \text{ m/s}$;
(c) It becomes internal energy.

P10.86 (a) 0.800 m/s^2 ; 0.400 m/s^2 ;
(b) 0.600 N between each cylinder and the plank; 0.200 N forward on each cylinder by the ground

P10.88 see the solution

P10.90 see the solution; to the left